CHAPTER 18

ELECTRICAL PROPERTIES

PROBLEM SOLUTIONS

Ohm's Law
Electrical Conductivity

18.1 (a) Compute the electrical conductivity of a 5.1-mm (0.2-in.) diameter cylindrical silicon specimen 51 mm (2 in.) long in which a current of 0.1 A passes in an axial direction. A voltage of 12.5 V is measured across two probes that are separated by 38 mm (1.5 in.).

(b) Compute the resistance over the entire 51 mm (2 in.) of the specimen.

Solution

This problem calls for us to compute the electrical conductivity and resistance of a silicon specimen.

(a) We use Equations 18.3 and 18.4 for the conductivity, as

$$\sigma = \frac{1}{\rho} = \frac{Il}{VA} = \frac{Il}{V\pi\left(\frac{d}{2}\right)^2}$$

And, incorporating values for the several parameters provided in the problem statement, leads to

$$\sigma = \frac{(0.1\ \text{A})(38 \times 10^{-3}\ \text{m})}{(12.5\ \text{V})(\pi)\left(\frac{5.1 \times 10^{-3}\ \text{m}}{2}\right)^2} = 14.9\ (\Omega\text{-m})^{-1}$$

(b) The resistance, R, may be computed using Equations 18.2 and 18.4, as

$$R = \frac{\rho l}{A} = \frac{1}{\sigma A} = \frac{1}{\sigma\pi\left(\frac{d}{2}\right)^2}$$

$$= \frac{51 \times 10^{-3}\ \text{m}}{\left[14.9\ (\Omega - \text{m})^{-1}\right](\pi)\left(\frac{5.1 \times 10^{-3}\ \text{m}}{2}\right)^2} = 168\ \Omega$$

18.2 A copper wire 100 m long must experience a voltage drop of less than 1.5 V when a current of 2.5 A passes through it. Using the data in Table 18.1, compute the minimum diameter of the wire.

Solution

For this problem, given that a copper wire 100 m long must experience a voltage drop of less than 1.5 V when a current of 2.5 A passes through it, we are to compute the minimum diameter of the wire. Combining Equations 18.3 and 18.4 and solving for the cross-sectional area A leads to

$$A = \frac{Il\rho}{V} = \frac{Il}{V\sigma}$$

From Table 18.1, for copper $\sigma = 6.0 \times 10^{7}\ (\Omega\text{-m})^{-1}$. Furthermore, inasmuch as $A = \pi\left(\frac{d}{2}\right)^2$ for a cylindrical wire, then

$$\pi\left(\frac{d}{2}\right)^2 = \frac{Il}{V\sigma}$$

or

$$d = \sqrt{\frac{4Il}{\pi V\sigma}}$$

When values for the several parameters given in the problem statement are incorporated into this expression, we get

$$d = \sqrt{\frac{(4)(2.5\ \text{A})(100\ \text{m})}{(\pi)(1.5\ \text{V})\left[6.0 \times 10^{7}\ (\Omega-\text{m})^{-1}\right]}}$$

$$= 1.88 \times 10^{-3}\ \text{m} = 1.88\ \text{mm}$$

18.3 An aluminum wire 4 mm in diameter is to offer a resistance of no more than 2.5 Ω. Using the data in Table 18.1, compute the maximum wire length.

Solution

This problem asks that we compute, for an aluminum wire 4 mm in diameter, the maximum length such that the resistance will not exceed 2.5 Ω. From Table 18.1 for aluminum, $\sigma = 3.8 \times 10^7$ $(\Omega\text{-m})^{-1}$. If d is the diameter then, combining Equations 18.2 and 18.4 leads to

$$l = \frac{RA}{\rho} = R\sigma A = R\sigma\pi\left(\frac{d}{2}\right)^2$$

$$= (2.5\ \Omega)\left[3.8 \times 10^7\ (\Omega - \text{m})^{-1}\right](\pi)\left(\frac{4 \times 10^{-3}\ \text{m}}{2}\right)^2 = 1194\ \text{m}$$

18.4 Demonstrate that the two Ohm's law expressions, Equations 18.1 and 18.5, are equivalent.

Solution

Let us demonstrate, by appropriate substitution and algebraic manipulation, that Equation 18.5 may be made to take the form of Equation 18.1. Now, Equation 18.5 is just

$$J = \sigma E$$

(In this equation we represent the electric field with an "E".) But, by definition, J is just the current density, the current per unit cross-sectional area, or $J = \frac{I}{A}$. Also, the electric field is defined by $E = \frac{V}{l}$. And, substituting these expressions into Equation 18.5 leads to

$$\frac{I}{A} = \sigma \frac{V}{l}$$

But, from Equations 18.2 and 18.4

$$\sigma = \frac{l}{RA}$$

and

$$\frac{I}{A} = \left(\frac{l}{RA}\right)\left(\frac{V}{l}\right)$$

Solving for V from this expression gives V = IR, which is just Equation 18.1.

18.5 (a) Using the data in Table 18.1, compute the resistance of a copper wire 3 mm (0.12 in.) in diameter and 2 m (78.7 in.) long. (b) What would be the current flow if the potential drop across the ends of the wire is 0.05 V? (c) What is the current density? (d) What is the magnitude of the electric field across the ends of the wire?

Solution

(a) In order to compute the resistance of this copper wire it is necessary to employ Equations 18.2 and 18.4. Solving for the resistance in terms of the conductivity,

$$R = \frac{\rho l}{A} = \frac{l}{\sigma A} = \frac{l}{\sigma \pi \left(\frac{d}{2}\right)^2}$$

From Table 18.1, the conductivity of copper is 6.0×10^7 $(\Omega\text{-m})^{-1}$, and

$$R = \frac{l}{\sigma \pi \left(\frac{d}{2}\right)^2} = \frac{2 \text{ m}}{\left[6.0 \times 10^7 \ (\Omega - \text{m})^{-1}\right](\pi)\left(\frac{3 \times 10^{-3} \text{ m}}{2}\right)^2}$$

$$= 4.7 \times 10^{-3} \ \Omega$$

(b) If V = 0.05 V then, from Equation 18.1

$$I = \frac{V}{R} = \frac{0.05 \text{ V}}{4.7 \times 10^{-3} \ \Omega} = 10.6 \ \text{ A}$$

(c) The current density is just

$$J = \frac{I}{A} = \frac{I}{\pi \left(\frac{d}{2}\right)^2} = \frac{10.6 \ \text{ A}}{\pi \left(\frac{3 \times 10^{-3} \text{ m}}{2}\right)^2} = 1.5 \times 10^6 \ \text{A/m}^2$$

(d) The electric field is just

$$E = \frac{V}{l} = \frac{0.05 \text{ V}}{2 \text{ m}} = 2.5 \times 10^{-2} \ \text{V/m}$$

Electronic and Ionic Conduction

18.6 What is the distinction between electronic and ionic conduction?

Solution

When a current arises from a flow of electrons, the conduction is termed electronic; for ionic conduction, the current results from the net motion of charged ions.

Energy Band Structures in Solids

18.7 How does the electron structure of an isolated atom differ from that of a solid material?

Solution

For an isolated atom, there exist discrete electron energy states (arranged into shells and subshells); each state may be occupied by, at most, two electrons, which must have opposite spins. On the other hand, an electron band structure is found for solid materials; within each band exist closely spaced yet discrete electron states, each of which may be occupied by, at most, two electrons, having opposite spins. The number of electron states in each band will equal the total number of corresponding states contributed by all of the atoms in the solid.

Conduction in Terms of Band and Atomic Bonding Models

18.8 In terms of electron energy band structure, discuss reasons for the difference in electrical conductivity between metals, semiconductors, and insulators.

Solution

For metallic materials, there are vacant electron energy states adjacent to the highest filled state; thus, very little energy is required to excite large numbers of electrons into conducting states. These electrons are those that participate in the conduction process, and, because there are so many of them, metals are good electrical conductors.

There are no empty electron states adjacent to and above filled states for semiconductors and insulators, but rather, an energy band gap across which electrons must be excited in order to participate in the conduction process. Thermal excitation of electrons will occur, and the number of electrons excited will be less than for metals, and will depend on the band gap energy. For semiconductors, the band gap is narrower than for insulators; consequently, at a specific temperature more electrons will be excited for semiconductors, giving rise to higher conductivities.

Electron Mobility

18.9 Briefly tell what is meant by the drift velocity and mobility of a free electron.

Solution

The drift velocity of a free electron is the average electron velocity in the direction of the force imposed by an electric field.

The mobility is the proportionality constant between the drift velocity and the electric field. It is also a measure of the frequency of scattering events (and is inversely proportional to the frequency of scattering).

18.10 (a) Calculate the drift velocity of electrons in germanium at room temperature and when the magnitude of the electric field is 1000 V/m. (b) Under these circumstances, how long does it take an electron to traverse a 25-mm (1-in.) length of crystal?

Solution

(a) The drift velocity of electrons in Ge may be determined using Equation 18.7. Since the room temperature mobility of electrons is 0.38 m^2/V-s (Table 18.3), and the electric field is 1000 V/m (as stipulated in the problem statement),

$$v_d = \mu_e E$$

$$= (0.38 \text{ m}^2/\text{V-s})(1000 \text{ V/m}) = 380 \text{ m/s}$$

(b) The time, t, required to traverse a given length, l (= 25 mm), is just

$$t = \frac{l}{v_d} = \frac{25 \times 10^{-3} \text{ m}}{380 \text{ m/s}} = 6.6 \times 10^{-5} \text{ s}$$

18.11 At room temperature the electrical conductivity and the electron mobility for copper are 6.0×10^7 $(\Omega\text{-m})^{-1}$ and 0.0030 m^2/V-s, respectively. (a) Compute the number of free electrons per cubic meter for copper at room temperature. (b) What is the number of free electrons per copper atom? Assume a density of 8.9 g/cm^3.

Solution

(a) The number of free electrons per cubic meter for copper at room temperature may be computed using Equation 18.8 as

$$n = \frac{\sigma}{|e|\,\mu_e}$$

$$= \frac{6.0 \times 10^7\,(\Omega - \text{m})^{-1}}{\left(1.602 \times 10^{-19}\ \text{C}\right)\left(0.003\ \text{m}^2/\text{V-s}\right)}$$

$$= 1.25 \times 10^{29}\ \text{m}^{-3}$$

(b) In order to calculate the number of free electrons per copper atom, we must first determine the number of copper atoms per cubic meter, N_{Cu}. From Equation 4.2 (and using the atomic weight value for Cu found inside the front cover—viz. 63.55 g/mol)

$$N_{Cu} = \frac{N_A\,\rho'}{A_{Cu}}$$

$$= \frac{\left(6.022 \times 10^{23}\ \text{atoms/mol}\right)\left(8.9\ \text{g/cm}^3\right)\left(10^6\ \text{cm}^3/\text{m}^3\right)}{63.55\ \text{g/mol}}$$

$$= 8.43 \times 10^{28}\ \text{m}^{-3}$$

(Note: in the above expression, density is represented by ρ' in order to avoid confusion with resistivity which is designated by ρ.) And, finally, the number of free electrons per aluminum atom is just n/N_{Cu}

$$\frac{n}{N_{Cul}} = \frac{1.25 \times 10^{29}\ \text{m}^{-3}}{8.43 \times 10^{28}\ \text{m}^{-3}} = 1.48$$

18.12 (a) Calculate the number of free electrons per cubic meter for gold assuming that there are 1.5 free electrons per gold atom. The electrical conductivity and density for Au are 4.3×10^7 $(\Omega\text{-m})^{-1}$ and 19.32 g/cm^3, respectively. (b) Now compute the electron mobility for Au.

Solution

(a) This portion of the problem asks that we calculate, for gold, the number of free electrons per cubic meter (n) given that there are 1.5 free electrons per gold atom, that the electrical conductivity is 4.3×10^7 $(\Omega\text{-m})^{-1}$, and that the density (ρ'_{Au}) is 19.32 g/cm^3. (Note: in this discussion, the density of silver is represented by ρ'_{Au} in order to avoid confusion with resistivity which is designated by ρ.) Since $n = 1.5N_{Au}$, and N_{Au} is defined in Equation 4.2 (and using the atomic weight of Au found inside the front cover—viz 196.97 g/mol), then

$$n = 1.5N_{Au} = 1.5\left[\frac{\rho'_{Au}N_A}{A_{Au}}\right]$$

$$= 1.5\left[\frac{(19.32\ \text{g/cm}^3)(6.022 \times 10^{23}\ \text{atoms/mol})}{196.97\ \text{g/mol}}\right]$$

$$= 8.86 \times 10^{22}\ \text{cm}^{-3} = 8.86 \times 10^{28}\ \text{m}^{-3}$$

(b) Now we are asked to compute the electron mobility, μ_e. Using Equation 18.8

$$\mu_e = \frac{\sigma}{n\,|e|}$$

$$= \frac{4.3 \times 10^7\ (\Omega - \text{m})^{-1}}{(8.86 \times 10^{28}\,\text{m}^{-3})(1.602 \times 10^{-19}\text{C})} = 3.03 \times 10^{-3}\ \text{m}^2/\text{V-s}$$

Electrical Resistivity of Metals

18.13 From Figure 18.38, estimate the value of A in Equation 18.11 for zinc as an impurity in copper–zinc alloys.

Solution

We want to solve for the parameter A in Equation 18.11 using the data in Figure 18.38. From Equation 18.11

$$A = \frac{\rho_i}{c_i (1 - c_i)}$$

However, the data plotted in Figure 18.38 is the total resistivity, ρ_{total}, and includes both impurity (ρ_i) and thermal (ρ_t) contributions (Equation 18.9). The value of ρ_t is taken as the resistivity at c_i = 0 in Figure 18.38, which has a value of 1.7×10^{-8} (Ω-m); this must be subtracted out. Below are tabulated values of A determined at c_i = 0.10, 0.20, and 0.30, including other data that were used in the computations. (Note: the c_i values were taken from the upper horizontal axis of Figure 18.38, since it is graduated in atom percent zinc.)

c_i	$1 - c_i$	ρ_{total} (Ω-m)	ρ_i (Ω-m)	A (Ω-m)
0.10	0.90	4.0×10^{-8}	2.3×10^{-8}	2.56×10^{-7}
0.20	0.80	5.4×10^{-8}	3.7×10^{-8}	2.31×10^{-7}
0.30	0.70	6.15×10^{-8}	4.45×10^{-8}	2.12×10^{-7}

So, there is a slight decrease of A with increasing c_i.

18.14 (a) Using the data in Figure 18.8, determine the values of ρ_0 and a from Equation 18.10 for pure copper. Take the temperature T to be in degrees Celsius. (b) Determine the value of A in Equation 18.11 for nickel as an impurity in copper, using the data in Figure 18.8. (c) Using the results of parts (a) and (b), estimate the electrical resistivity of copper containing 1.75 at% Ni at 100°C.

Solution

(a) Perhaps the easiest way to determine the values of ρ_0 and a in Equation 18.10 for pure copper in Figure 18.8, is to set up two simultaneous equations using two resistivity values (labeled ρ_{t1} and ρ_{t2}) taken at two corresponding temperatures (T_1 and T_2). Thus,

$$\rho_{t1} = \rho_0 + aT_1$$

$$\rho_{t2} = \rho_0 + aT_2$$

And solving these equations simultaneously lead to the following expressions for a and ρ_0:

$$a = \frac{\rho_{t1} - \rho_{t2}}{T_1 - T_2}$$

$$\rho_0 = \rho_{t1} - T_1\left[\frac{\rho_{t1} - \rho_{t2}}{T_1 - T_2}\right]$$

$$= \rho_{t2} - T_2\left[\frac{\rho_{t1} - \rho_{t2}}{T_1 - T_2}\right]$$

From Figure 18.8, let us take T_1 = –150°C, T_2 = –50°C, which gives $\rho_{t1} = 0.6 \times 10^{-8}$ (Ω-m), and $\rho_{t2} = 1.25 \times 10^{-8}$ (Ω-m). Therefore

$$a = \frac{\rho_{t1} - \rho_{t2}}{T_1 - T_2}$$

$$= \frac{\left[(0.6 \times 10^{-8}) - (1.25 \times 10^{-8})\right](\Omega\text{-m})}{-150\,°\text{C} - (-50°\text{C})}$$

$$6.5 \times 10^{-11}\ (\Omega\text{-m})/°\text{C}$$

and

$$\rho_0 = \rho_{t1} - T_1\left[\frac{\rho_{t1} - \rho_{t2}}{T_1 - T_2}\right]$$

$$= (0.6 \times 10^{-8}) - (-150)\ \frac{\left[(0.6 \times 10^{-8}) - (1.25 \times 10^{-8})\right](\Omega\text{-m})}{-150\,°\text{C} - (-50°\text{C})}$$

$$= 1.58 \times 10^{-8}\ (\Omega\text{-m})$$

(b) For this part of the problem, we want to calculate A from Equation 18.11

$$\rho_i = Ac_i\,(1 - c_i)$$

In Figure 18.8, curves are plotted for three c_i values (0.0112, 0.0216, and 0.0332). Let us find A for each of these c_i's by taking a ρ_{total} from each curve at some temperature (say 0°C) and then subtracting out ρ_i for pure copper at this same temperature (which is 1.7×10^{-8} Ω-m). Below is tabulated values of A determined from these three c_i values, and other data that were used in the computations.

c_i	$1 - c_i$	ρ_{total} (Ω-m)	ρ_i (Ω-m)	A (Ω-m)
0.0112	0.989	3.0×10^{-8}	1.3×10^{-8}	1.17×10^{-6}
0.0216	0.978	4.2×10^{-8}	2.5×10^{-8}	1.18×10^{-6}
0.0332	0.967	5.5×10^{-8}	3.8×10^{-8}	1.18×10^{-6}

The average of these three A values is 1.18×10^{-6} (Ω-m).

(c) We use the results of parts (a) and (b) to estimate the electrical resistivity of copper containing 1.75 at% Ni ($c_i = 0.0175$) at 100°C. The total resistivity is just

$$\rho_{total} = \rho_t + \rho_i$$

Or incorporating the expressions for ρ_t and ρ_i from Equations 18.10 and 18.11, and the values of ρ_0, a, and A determined above, leads to

$$\rho_{total} = (\rho_0 + aT) + Ac_i\,(1 - c_i)$$

$$= \left\{1.58 \times 10^{-8} (\Omega\text{-m}) + \left[6.5 \times 10^{-11} (\Omega\text{-m})/°\text{C}\right](100\,°\text{C})\right\}$$

$$+ \left\{\left[1.18 \times 10^{-6} (\Omega\text{-m})\right](0.0175)(1 - 0.0175)\right\}$$

$$= 4.25 \times 10^{-8} (\Omega\text{-m})$$

18.15 Determine the electrical conductivity of a Cu-Ni alloy that has a yield strength of 125 MPa (18,000 psi). You will find Figure 7.16 helpful.

Solution

We are asked to determine the electrical conductivity of a Cu-Ni alloy that has a yield strength of 125 MPa. From Figure 7.16b, the composition of an alloy having this tensile strength is about 20 wt% Ni. For this composition, the resistivity is about 27×10^{-8} Ω-m (Figure 18.9). And since the conductivity is the reciprocal of the resistivity, Equation 18.4, we have

$$\sigma = \frac{1}{\rho} = \frac{1}{27 \times 10^{-8}\ \Omega - \mathrm{m}} = 3.70 \times 10^{6}\ (\Omega\text{ - m})^{-1}$$

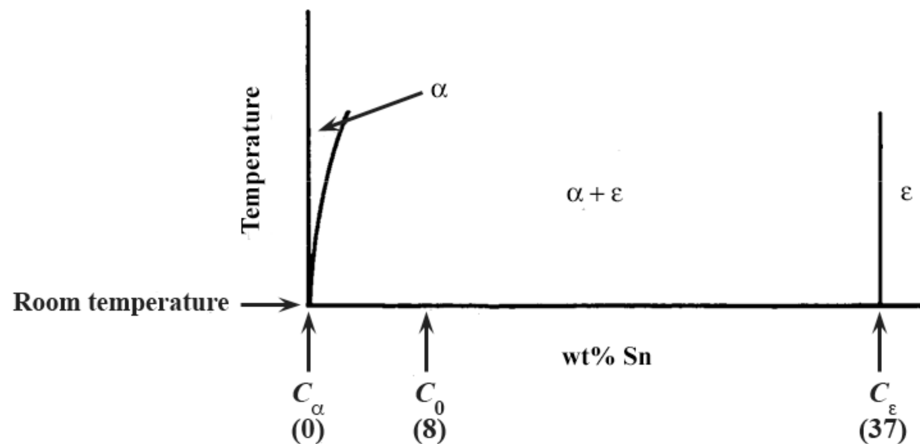
18.16 Tin bronze has a composition of 92 wt% Cu and 8 wt% Sn, and consists of two phases at room temperature: an α phase, which is copper containing a very small amount of tin in solid solution, and an ε phase, which consists of approximately 37 wt% Sn. Compute the room temperature conductivity of this alloy given the following data:

Phase	Electrical Resistivity (Ω-m)	Density (g/cm^3)
α	1.88×10^{-8}	8.94
ε	5.32×10^{-7}	8.25

Solution

This problem asks for us to compute the room-temperature conductivity of a two-phase Cu-Sn alloy which composition is 92 wt% Cu-8 wt% Sn. It is first necessary for us to determine the volume fractions of the α and ε phases, after which the resistivity (and subsequently, the conductivity) may be calculated using Equation 18.12. Weight fractions of the two phases are first calculated using the phase diagram information provided in the problem.

We may represent a portion of the phase diagram near room temperature as follows:



Applying the lever rule to this situation

$$W_\alpha = \frac{C_\varepsilon - C_0}{C_\varepsilon - C_\alpha} = \frac{37 - 8}{37 - 0} = 0.784$$

$$W_\varepsilon = \frac{C_0 - C_\alpha}{C_\varepsilon - C_\alpha} = \frac{8 - 0}{37 - 0} = 0.216$$

We must now convert these mass fractions into volume fractions using the phase densities given in the problem statement. (Note: in the following expressions, density is represented by ρ' in order to avoid confusion with resistivity which is designated by ρ.) Utilization of Equations 9.6a and 9.6b leads to

$$V_\alpha = \frac{\dfrac{W_\alpha}{\rho'_\alpha}}{\dfrac{W_\alpha}{\rho'_\alpha} + \dfrac{W_\varepsilon}{\rho'_\varepsilon}}$$

$$= \frac{\dfrac{0.784}{8.94 \text{ g/cm}^3}}{\dfrac{0.784}{8.94 \text{ g/cm}^3} + \dfrac{0.216}{8.25 \text{ g/cm}^3}}$$

$$= 0.770$$

$$V_\varepsilon = \frac{\dfrac{W_\varepsilon}{\rho'_\varepsilon}}{\dfrac{W_\alpha}{\rho'_\alpha} + \dfrac{W_\varepsilon}{\rho'_\varepsilon}}$$

$$= \frac{\dfrac{0.216}{8.25 \text{ g/cm}^3}}{\dfrac{0.784}{8.94 \text{ g/cm}^3} + \dfrac{0.216}{8.25 \text{ g/cm}^3}}$$

$$= 0.230$$

Now, using Equation 18.12

$$\rho = \rho_\alpha V_\alpha + \rho_\varepsilon V_\varepsilon$$

$$= \left(1.88 \times 10^{-8} \ \Omega\text{-m}\right)(0.770) + \left(5.32 \times 10^{-7} \ \Omega\text{-m}\right)(0.230)$$

$$= 1.368 \times 10^{-7} \ \Omega\text{-m}$$

Finally, for the conductivity (Equation 18.4)

$$\sigma = \frac{1}{\rho} = \frac{1}{1.368 \times 10^{-7}\ \Omega - \text{m}} = 7.31 \times 10^{6}\ (\Omega\text{ - m})^{-1}$$

18.17 A cylindrical metal wire 2 mm (0.08 in.) in diameter is required to carry a current of 10 A with a minimum of 0.03 V drop per foot (300 mm) of wire. Which of the metals and alloys listed in Table 18.1 are possible candidates?

Solution

We are asked to select which of several metals may be used for a 2 mm diameter wire to carry 10 A, and have a voltage drop less than 0.03 V per foot (300 mm). Using Equations 18.3 and 18.4, let us determine the minimum conductivity required, and then select from Table 18.1, those metals that have conductivities greater than this value. Combining Equations 18.3 and 18.4, the minimum conductivity is just

$$\sigma = \frac{1}{\rho} = \frac{Il}{VA} = \frac{Il}{V\pi\left(\frac{d}{2}\right)^2}$$

$$= \frac{(10\ \text{A})(300 \times 10^{-3}\ \text{m})}{(0.03\ \text{V})\ (\pi)\left(\frac{2 \times 10^{-3}\ \text{m}}{2}\right)^2} = 3.2 \times 10^7\ (\Omega\text{-m})^{-1}$$

Thus, from Table 18.1, only aluminum, gold, copper, and silver are candidates.

Intrinsic Semiconduction

18.18 (a) Using the data presented in Figure 18.16, determine the number of free electrons per atom for intrinsic germanium and silicon at room temperature (298 K). The densities for Ge and Si are 5.32 and 2.33 g/cm^3, respectively.

(b) Now explain the difference in these free-electron-per-atom values.

Solution

(a) For this part of the problem, we first read, from Figure 18.16, the number of free electrons (i.e., the intrinsic carrier concentration) at room temperature (298 K). These values are $n_i(\mathrm{Ge}) = 5 \times 10^{19}$ m^{-3} and $n_i(\mathrm{Si}) = 7 \times 10^{16}$ m^{-3}.

Now, the number of atoms per cubic meter for Ge and Si (N_{Ge} and N_{Si}, respectively) may be determined using Equation 4.2 which involves the densities (ρ'_{Ge} and ρ'_{Si}) and atomic weights (A_{Ge} and A_{Si}). (Note: here we use ρ' to represent density in order to avoid confusion with resistivity, which is designated by ρ. Also, the atomic weights for Ge and Si, 72.64 and 28.09 g/mol, respectively, are found inside the front cover.) Therefore,

$$N_{Ge} = \frac{N_A \rho'_{Ge}}{A_{Ge}}$$

$$= \frac{(6.022 \times 10^{23} \text{ atoms/mol})(5.32 \text{ g/cm}^3)(10^6 \text{ cm}^3/\text{m}^3)}{72.64 \text{ g/mol}}$$

$$= 4.41 \times 10^{28} \text{ atoms/m}^3$$

Similarly, for Si

$$N_{Si} = \frac{N_A \rho'_{Si}}{A_{Si}}$$

$$= \frac{(6.022 \times 10^{23} \text{ atoms/mol})(2.33 \text{ g/cm}^3)(10^6 \text{ cm}^3/\text{m}^3)}{28.09 \text{ g/mol}}$$

$$= 5.00 \times 10^{28} \text{ atoms/m}^3$$

Finally, the ratio of the number of free electrons per atom is calculated by dividing n_i by N. For Ge

$$\frac{n_i(\mathrm{Ge})}{N_{\mathrm{Ge}}} = \frac{5 \times 10^{19} \text{ electrons/m}^3}{4.41 \times 10^{28} \text{ atoms/m}^3}$$

$$1.13 \times 10^{-9} \text{ electron/atom}$$

And, for Si

$$\frac{n_i(\mathrm{Si})}{N_{\mathrm{Si}}} = \frac{7 \times 10^{16} \text{ electrons/m}^3}{5.00 \times 10^{28} \text{ atoms/m}^3}$$

$$= 1.40 \times 10^{-12} \text{ electron/atom}$$

(b) The difference is due to the magnitudes of the band gap energies (Table 18.3). The band gap energy at room temperature for Si (1.11 eV) is larger than for Ge (0.67 eV), and, consequently, the probability of excitation across the band gap for a valence electron is much smaller for Si.

18.19 For intrinsic semiconductors, the intrinsic carrier concentration n_i depends on temperature as follows:

$$n_i \propto \exp\left(-\frac{E_g}{2kT}\right) \qquad (18.35a)$$

or taking natural logarithms,

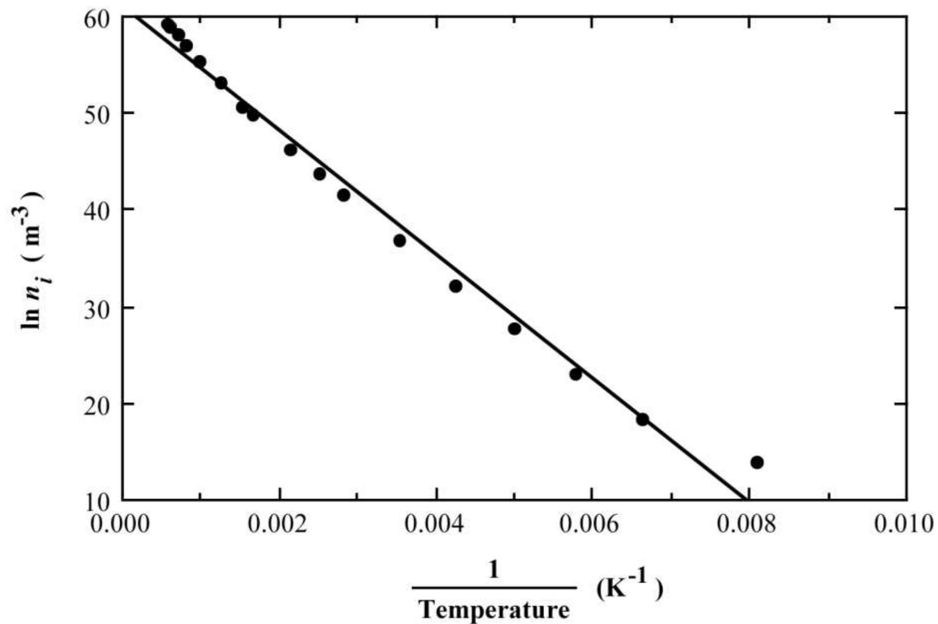
$$\ln n_i \propto -\frac{E_g}{2kT} \qquad (18.35b)$$

Thus, a plot of ln n_i versus 1/T $(K)^{-1}$ should be linear and yield a slope of $-E_g/2k$. Using this information and the data presented in Figure 18.16, determine the band gap energies for silicon and germanium, and compare these values with those given in Table 18.3.

Solution

This problem asks that we make plots of ln n_i versus reciprocal temperature for both Si and Ge, using the data presented in Figure 18.16, and then determine the band gap energy for each material realizing that the slope of the resulting line is equal to $-E_g/2k$.

Below is shown such a plot for Si.



The slope of the line is equal to

$$\text{Slope} = \frac{\Delta \ln \eta_i}{\Delta\left(\frac{1}{T}\right)} = \frac{\ln \eta_1 - \ln \eta_2}{\frac{1}{T_1} - \frac{1}{T_2}}$$

Let us take $1/T_1$ = 0.001 and $1/T_2$ └ └‼◄‼→ └└WKHLUFRUUHVSRQGLQJ └Q└η values are ln η_1 = 54.80 and ln η_2 = 16.00. Incorporating these values into the above expression leads to a slope of

$$\text{Slope} = \frac{54.80 - 16.00}{0.001 - 0.007} = -6467$$

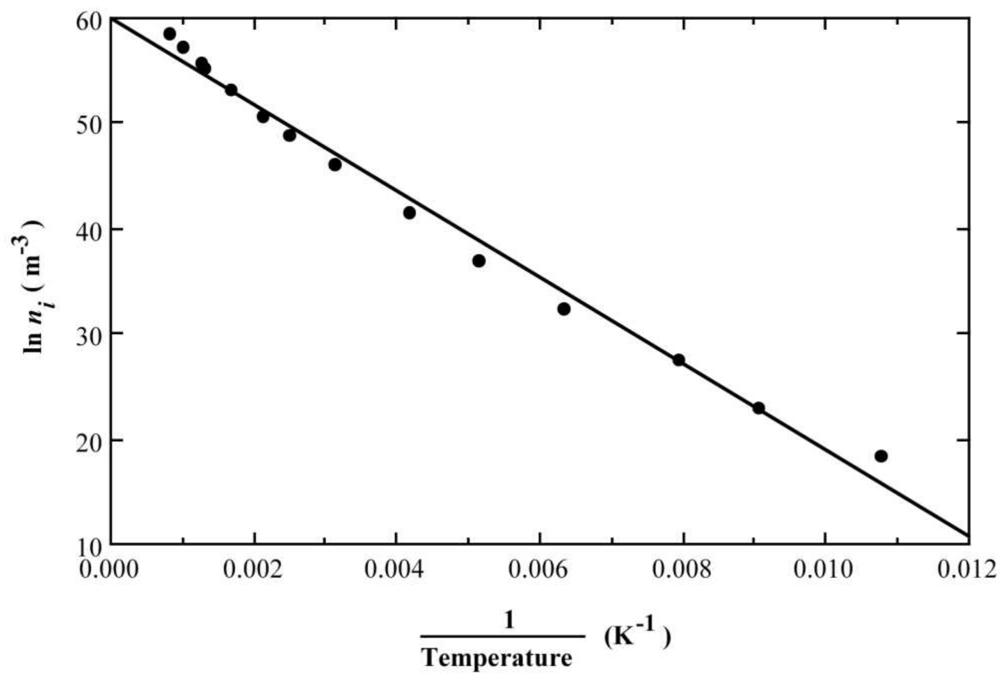
This slope leads to an E_g value of

$$E_g = -2k\ (\text{Slope})$$

$$= -2\left(8.62 \times 10^{-5}\ \text{eV/K}\right)(-6467) = 1.115\ \text{eV}$$

The value cited in Table 18.3 is 1.11 eV.

Now for Ge, an analogous plot is shown below.



We calculate the slope and band gap energy values in the manner outlined above. Let us take $1/T_1$ = 0.001 and $1/T_2$ = 0.‼¶¶ └└WKHLU┘FRUUHVSRQGLQJ └OQ└η values are ln η_1 = 55.56 and ln η_2 = 14.80. Incorporating these values into the above expression leads to a slope of

$$\text{Slope} = \frac{55.56 - 14.80}{0.001 - 0.011} = -4076$$

This slope leads to an E_g value of

$$E_g = -2k\,(\text{Slope})$$

$$= -2\left(8.62 \times 10^{-5}\ \text{eV/K}\right)(-4076) = 0.70\ \text{eV}$$

This value is in good agreement with the 0.67 eV cited in Table 18.3.

18.20 Briefly explain the presence of the factor 2 in the denominator of Equation 18.35a.

Solution

The factor 2 in Equation 18.35a takes into account the creation of two charge carriers (an electron and a hole) for each valence-band-to-conduction-band intrinsic excitation; both charge carriers may participate in the conduction process.

18.21 At room temperature the electrical conductivity of PbTe is 500 $(\Omega\text{-m})^{-1}$, whereas the electron and hole mobilities are 0.16 and 0.075 $m^2/V\text{-}s$, respectively. Compute the intrinsic carrier concentration for PbTe at room temperature.

Solution

In this problem we are asked to compute the intrinsic carrier concentration for PbTe at room temperature. Since the conductivity and both electron and hole mobilities are provided in the problem statement, all we need do is solve for n and p (i.e., n_i) using Equation 18.15. Thus,

$$n_i = \frac{\sigma}{|e|(\mu_e + \mu_h)}$$

$$= \frac{500\ (\Omega - \text{m})^{-1}}{(1.602 \times 10^{-19}\ \text{C})(0.16 + 0.075)\ \text{m}^2/\text{V - s}}$$

$$= 1.33 \times 10^{22}\ \text{m}^{-3}$$

18.22 Is it possible for compound semiconductors to exhibit intrinsic behavior? Explain your answer.

Solution

Yes, compound semiconductors can exhibit intrinsic behavior. They will be intrinsic even though they are composed of two different elements as long as the electrical behavior is not influenced by the presence of other elements.

18.23 For each of the following pairs of semiconductors, decide which will have the smaller band gap energy, E_g, and then cite the reason for your choice. (a) ZnS and CdSe, (b) Si and C (diamond), (c) Al_2O_3 and ZnTe, (d) InSb and ZnSe, and (e) GaAs and AlP.

Solution

This problem calls for us to decide for each of several pairs of semiconductors, which will have the smaller band gap energy and then cite a reason for the choice.

(a) Cadmium selenide will have a smaller band gap energy than zinc sulfide. Both are II-VI compounds, and Cd and Se are both lower vertically in the periodic table (Figure 2.6) than Zn and S. In moving from top to bottom down the periodic table, E_g decreases.

(b) Silicon will have a smaller band gap energy than diamond since Si is lower in column IVA of the periodic table than is C.

(c) Zinc telluride will have a smaller band gap energy that aluminum oxide. There is a greater disparity between the electronegativities for aluminum and oxygen [1.5 versus 3.5 (Figure 2.7)] than for zinc and tellurium (1.6 and 2.1). For binary compounds, the larger the difference between the electronegativities of the elements, the greater the band gap energy.

(d) Indium antimonide will have a smaller band gap energy than zinc selenide. These materials are III-V and II-VI compounds, respectively; Thus, in the periodic table, In and Sb are closer together horizontally than are Zn and Se. Furthermore, both In and Sb reside below Zn and Se in the periodic table.

(e) Gallium arsenide will have a smaller band gap energy than aluminum phosphide. Both are III-V compounds, and Ga and As are both lower vertically in the periodic table than Al and P.

Extrinsic Semiconduction

18.24 Define the following terms as they pertain to semiconducting materials: intrinsic, extrinsic, compound, elemental. Now provide an example of each.

Solution

These semiconductor terms are defined in the Glossary. Examples are as follows: intrinsic--high purity (undoped) Si, GaAs, CdS, etc.; extrinsic--P-doped Ge, B-doped Si, S-doped GaP, etc.; compound--GaAs, InP, CdS, etc.; elemental--Ge and Si.

18.25 An n-type semiconductor is known to have an electron concentration of 3×10^{18} m^{-3}. If the electron drift velocity is 100 m/s in an electric field of 500 V/m, calculate the conductivity of this material.

Solution

The conductivity of this material may be computed using Equation 18.16. But before this is possible, it is necessary to calculate the value of μ_e from Equation 18.7. Thus, the electron mobility is equal to

$$\mu_e = \frac{v_d}{E}$$

$$= \frac{100 \text{ m/s}}{500 \text{ V/m}} = 0.20 \text{ m}^2/\text{V}-\text{s}$$

Thus, from Equation 18.16, the conductivity is

$$\sigma = n|e|\mu_e$$

$$= (3 \times 10^{18} \text{ m}^{-3})(1.602 \times 10^{-19} \text{C})(0.20 \text{ m}^2/\text{V}-\text{s})$$

$$= 0.096 \ (\Omega\text{-m})^{-1}$$

18.26 (a) In your own words, explain how donor impurities in semiconductors give rise to free electrons in numbers in excess of those generated by valence band–conduction band excitations. (b) Also explain how acceptor impurities give rise to holes in numbers in excess of those generated by valence band–conduction band excitations.

The explanations called for are found in Section 18.11.

18.27 (a) Explain why no hole is generated by the electron excitation involving a donor impurity atom. (b) Explain why no free electron is generated by the electron excitation involving an acceptor impurity atom.

Solution

(a) No hole is generated by an electron excitation involving a donor impurity atom because the excitation comes from a level within the band gap, and thus, no missing electron is created within the normally filled valence band.

(b) No free electron is generated by an electron excitation involving an acceptor impurity atom because the electron is excited from the valence band into the impurity level within the band gap; no free electron is introduced into the conduction band.

18.28 Will each of the following elements act as a donor or an acceptor when added to the indicated semiconducting material? Assume that the impurity elements are substitutional.

Impurity	**Semiconductor**
P	Ge
S	AlP
In	CdTe
Al	Si
Cd	GaAs
Sb	ZnSe

Solution

Phosphorus will act as a donor in Ge. Since it (P) is from group VA of the periodic table (Figure 2.6), a P atom has one more valence electron than a Ge atom.

Sulfur will act as a donor in AlP. Since S is from group VIA of the periodic table, it will substitute for P; also, an S atom has one more valence electron than a P atom.

Indium will act as a donor in CdTe. Since In is from group IIIA of the periodic table, it will substitute for Cd; furthermore, an In atom has one more valence electron than a Cd atom.

Aluminum will act as an acceptor in Si. Since it (Al) is from group IIIA of the periodic table (Figure 2.6), an Al atom has one less valence electron than a Si atom.

Cadmium will act as an acceptor in GaAs. Since Cd is from group IIB of the periodic table, it will substitute for Ga; furthermore, a Cd atom has one less valence electron than a Ga atom.

Antimony will act as an acceptor in ZnSe. Since Sb is from group VA of the periodic table, it will substitute for Se; and, an Sb atom has one less valence electron than an Se atom.

18.29 (a) The room-temperature electrical conductivity of a silicon specimen is 5.93×10^{-3} $(\Omega\text{-m})^{-1}$. The hole concentration is known to be 7.0×10^{17} m^{-3}. Using the electron and hole mobilities for silicon in Table 18.3, compute the electron concentration. (b) On the basis of the result in part (a), is the specimen intrinsic, n-type extrinsic, or p-type extrinsic? Why?

Solution

(a) In this problem, for a Si specimen, we are given values for p $(7.0 \times 10^{17}\ m^{-3})$ and $\sigma\,[5.93 \times 10^{-3}\ (\Omega\text{-m})^{-1}]$, while values for μ_h and μ_e (0.05 and 0.14 m^2/V-s, respectively) are found in Table 18.3. In order to solve for n we must use Equation 18.13, which, after rearrangement, leads to

$$n = \frac{\sigma - p|e|\mu_h}{|e|\mu_e}$$

$$= \frac{5.93 \times 10^{-3}\ (\Omega - m)^{-1} - (7.0 \times 10^{17}\ m^{-3})(1.602 \times 10^{-19}\ C)(0.05\ m^2/V\text{-}s)}{(1.602 \times 10^{-19}\ C)(0.14\ m^2/V\text{-}s)}$$

$$= 1.44 \times 10^{16}\ m^{-3}$$

(b) This material is p-type extrinsic since p $(7.0 \times 10^{17}\ m^{-3})$ is greater than n $(1.44 \times 10^{16}\ m^{-3})$.

18.30 Germanium to which 5×10^{22} m^{-3} Sb atoms have been added is an extrinsic semiconductor at room temperature, and virtually all the Sb atoms may be thought of as being ionized (i.e., one charge carrier exists for each Sb atom). (a) Is this material n-type or p-type? (b) Calculate the electrical conductivity of this material, assuming electron and hole mobilities of 0.1 and 0.05 m^2/V-s, respectively.

Solution

(a) (a) This germanium material to which has been added 5×10^{22} m^{-3} Sb atoms is n-type since Sb is a donor in Ge. (Antimony is from group VA of the periodic table--Ge is from group IVA.)

(b) Since this material is n-type extrinsic, Equation 18.16 is valid. Furthermore, each Sb will donate a single electron, or the electron concentration is equal to the Sb concentration since all of the Sb atoms are ionized at room temperature; that is $n = 5 \times 10^{22}$ m^{-3}, and, as given in the problem statement, $\mu_e = 0.1$ m^2/V-s. Thus

$$\sigma = n|e|\mu_e$$

$$= (5 \times 10^{22} \text{ m}^{-3})(1.602 \times 10^{-19} \text{ C})(0.1 \text{ m}^2/\text{V-s})$$

$$= 800 \ (\Omega\text{-m})^{-1}$$

18.31 The following electrical characteristics have been determined for both intrinsic and p-type extrinsic indium phosphide (InP) at room temperature:

	σ (Ω -m)$^{-1}$	**n (m^{-3})**	**p (m^{-3})**
Intrinsic	2.5×10^{-6}	3.0×10^{13}	3.0×10^{13}
Extrinsic (n-type)	3.6×10^{-5}	4.5×10^{14}	2.0×10^{12}

Calculate electron and hole mobilities.

Solution

In order to solve for the electron and hole mobilities for InP, we must write conductivity expressions for the two materials, of the form of Equation 18.13—i.e.,

$$\sigma = n|e|\mu_e + p|e|\mu_h$$

For the intrinsic material

$$2.5 \times 10^{-6} (\Omega - \text{m})^{-1} = (3.0 \times 10^{13} \text{ m}^{-3})(1.602 \times 10^{-19} \text{ C})\mu_e$$
$$+ (3.0 \times 10^{13} \text{ m}^{-3})(1.602 \times 10^{-19} \text{ C})\mu_h$$

which reduces to

$$0.52 = \mu_e + \mu_h$$

Whereas, for the extrinsic InP

$$3.6 \times 10^{-5} (\Omega - \text{m})^{-1} = (4.5 \times 10^{14} \text{ m}^{-3})(1.602 \times 10^{-19} \text{ C})\mu_e$$
$$+ (2.0 \times 10^{12} \text{ m}^{-3})(1.602 \times 10^{-19} \text{ C})\mu_h$$

which may be simplified to

$$112.4 = 225\mu_e + \mu_h$$

Thus, we have two independent expressions with two unknown mobilities. Upon solving these equations simultaneously, we get $\mu_e = 0.50$ m^2/V-s and $\mu_h = 0.02$ m^2/V-s.

The Temperature Dependence of Carrier Concentration

18.32 Calculate the conductivity of intrinsic silicon at 100°C.

Solution

In order to estimate the electrical conductivity of intrinsic silicon at 100°C, we must employ Equation 18.15. However, before this is possible, it is necessary to determine values for n_i, μ_e, and μ_h. According to Figure 18.16, at 100°C (373 K), $n_i = 2 \times 10^{18}$ m^{-3}, whereas from the "$<10^{20}$ m^{-3}" curves of Figures 18.19a and 18.19b, at 100ºC (373 K), $\mu_e = 0.09$ m^2/V-s and $\mu_h = 0.032$ m^2/V-s (realizing that the mobility axes of these two plot are scaled logarithmically). Thus, the conductivity at 100°C is

$$\sigma = n_i |e| (\mu_e + \mu_h)$$

$$\sigma = (2 \times 10^{18}\ \mathrm{m^{-3}})(1.602 \times 10^{-19}\ \mathrm{C})(0.09\ \mathrm{m^2/V\text{-}s} + 0.032\ \mathrm{m^2/V\text{-}s})$$

$$= 0.039\ (\Omega\text{-m})^{-1}$$

18.33 At temperatures near room temperature, the temperature dependence of the conductivity for intrinsic germanium is found to equal

$$\sigma = CT^{-3/2} \exp\left(-\frac{E_g}{2kT}\right) \quad (18.36)$$

where C is a temperature-independent constant and T is in Kelvins. Using Equation 18.36, calculate the intrinsic electrical conductivity of germanium at 150°C.

Solution

It first becomes necessary to solve for C in Equation 18.36 using the room-temperature (298 K) conductivity [2.2 $(\Omega\text{-m})^{-1}$] (Table 18.3). This is accomplished by taking natural logarithms of both sides of Equation 18.36 as

$$\ln \sigma = \ln C - \frac{3}{2}\ln T - \frac{E_g}{2kT}$$

and after rearranging and substitution of values for E_g (0.67 eV, Table 18.3), and the room-temperature conductivity, we get

$$\ln C = \ln \sigma + \frac{3}{2}\ln T + \frac{E_g}{2kT}$$

$$= \ln (2.2) + \frac{3}{2}\ln (298) + \frac{0.67 \text{ eV}}{(2)(8.62 \times 10^{-5} \text{ eV/K})(298 \text{ K})}$$

$$= 22.38$$

Now, again using Equation 18.36, we are able to compute the conductivity at 423 K (150°C)

$$\ln \sigma = \ln C - \frac{3}{2}\ln T - \frac{E_g}{2kT}$$

$$= 22.38 - \frac{3}{2}\ln (423 \text{ K}) - \frac{0.67 \text{ eV}}{(2)(8.62 \times 10^{-5} \text{ eV/K})(423 \text{ K})}$$

$$= 4.12$$

which leads to

$$\sigma = e^{4.12} = 61.6\ (\Omega\text{-m})^{-1}.$$

18.34 Using Equation 18.36 and the results of Problem 18.33, determine the temperature at which the electrical conductivity of intrinsic germanium is 22.8 $(\Omega\text{-m})^{-1}$.

Solution

This problem asks that we determine the temperature at which the electrical conductivity of intrinsic Ge is 22.8 $(\Omega\text{-m})^{-1}$, using Equation 18.36 and the results of Problem 18.33. First of all, taking logarithms of Equation 18.36

$$\ln \sigma = \ln C - \frac{3}{2} \ln T - \frac{E_g}{2kT}$$

And, from Problem 18.33 the value of ln C was determined to be 22.38. Using this and $\sigma = 22.8\ (\Omega\text{-m})^{-1}$, the above equation takes the form

$$\ln 22.8 = 22.38 - \frac{3}{2} \ln T - \frac{0.67\ \text{eV}}{(2)(8.62 \times 10^{-5}\ \text{eV/K})(T)}$$

In order to solve for T from the above expression it is necessary to use an equation solver. For some solvers, the following set of instructions may be used:

ln(22.8) = 22.38 –1.5*ln(T) – 0.67/(2*8.62*10^-5*T)

The resulting solution is T└ ┌→↑☼ZKLFK YDOXH LV WKH WHP SHUDWXUH LQ. WKLV FRUUHVSRQGV WR T(ºC) = 375 – 273 = 102°C.

18.35 Estimate the temperature at which GaAs has an electrical conductivity of 3.7 3 10^{23} $(\text{V-m})^{21}$ assuming the temperature dependence for σ of Equation 18.36. The data shown in Table 18.3 might prove helpful.

Solution

This problem asks that we estimate the temperature at which GaAs has an electrical conductivity of 3.7×10^{-3} $(\Omega\text{-m})^{-1}$ assuming that the conductivity has a temperature dependence as shown in Equation 18.36. From the room temperature (298 K) conductivity [10^{-6} $(\Omega\text{-m})^{-1}$] and band gap energy (1.42 eV) of Table 18.3 we determine the value of C (Equation 18.36) by taking natural logarithms of both sides of the equation, and after rearrangement as follows:

$$\ln C = \ln \sigma + \frac{3}{2}\ln T + \frac{E_g}{2kT}$$

$$= \ln\left[10^{-6}\ (\Omega - \text{m})^{-1}\right] + \frac{3}{2}\ln\ (298\ \text{K}) + \frac{1.42\ \text{eV}}{(2)\left(8.62 \times 10^{-5}\ \text{eV/K}\right)(298\ \text{K})}$$

$$= 22.37$$

Now we substitute this value into Equation 18.36 in order to determine the value of T for which $\sigma = 3.7 \times 10^{-3}$ $(\Omega\text{-m})^{-1}$, thus

$$\ln \sigma = \ln C - \frac{3}{2}\ln T - \frac{E_g}{2kT}$$

$$\ln\left[3.7 \times 10^{-3}\ (\Omega\text{ - m})^{-1}\right] = 22.37 - \frac{3}{2}\ln T - \frac{1.42\ \text{eV}}{(2)\left(8.62 \times 10^{-5}\ \text{eV/K}\right)(T)}$$

This equation may be solved for T using an equation solver. For some solvers, the following set of instructions may be used:

```
ln(3.7*10^–3) = 22.37 – 1.5*ln(T) – 1.42/(2*8.62*10^–5*T)
```

The resulting solution is T└ ╎┬→ └WKLV└YDOXH└LV└WKH└WHPSHUDWXUH└LQ.└ └ZKLFK└FRUUHVSRQGV└WR└T(ºC) = 437 K – 273 = 164°C.

18.36 Compare the temperature dependence of the conductivity for metals and intrinsic semiconductors. Briefly explain the difference in behavior.

Solution

For metals, the temperature dependence is described by Equation 18.10 (and converting from resistivity to conductivity using Equation 18.4), as

$$\sigma = \frac{1}{\rho_0 + aT}$$

That is, the electrical conductivity decreases with increasing temperature.

Alternatively, from Equation 18.8, the conductivity of metals is equal to

$$\sigma = n|e|\mu_e$$

As the temperature rises, n will remain virtually constant, whereas the mobility (μ_e) will decrease, because the thermal scattering of free electrons will become more efficient. Since |e| is independent of temperature, the net result will be diminishment in the magnitude of σ.

For intrinsic semiconductors, the temperature-dependence of conductivity is just the opposite of that for metals—i.e, conductivity increases with rising temperature. One explanation is as follows: Equation 18.15 describes the conductivity; i.e.,

$$\sigma = n|e|(\mu_e + \mu_h) = p|e|(\mu_e + \mu_h)$$
$$= n_i|e|(\mu_e + \mu_h)$$

Both n and p increase dramatically with rising temperature (Figure 18.16), since more thermal energy becomes available for valence band-conduction band electron excitations. The magnitudes of μ_e and μ_h will diminish somewhat with increasing temperature (per the upper curves of Figures 18.19a and 18.19b), as a consequence of the thermal scattering of electrons and holes. However, this reduction of μ_e and μ_h will be overwhelmed by the increase in n and p, with the net result is that σ increases with temperature.

An alternative explanation is as follows: for an intrinsic semiconductor the temperature dependence is represented by an equation of the form of Equation 18.36. This expression contains two terms that involve temperature—a preexponential one (in this case $T^{-3/2}$) and the other in the exponential. With rising temperature the preexponential term decreases, while the exp $(-E_g/2kT)$ parameter increases. With regard to relative magnitudes, the

exponential term increases much more rapidly than the preexponential one, such that the electrical conductivity of an intrinsic semiconductor increases with rising temperature.

Factors That Affect Carrier Mobility

18.37 Calculate the room-temperature electrical conductivity of silicon that has been doped with 5×10^{22} m^{-3} of boron atoms.

Solution

This problems asks that we determine the room-temperature electrical conductivity of silicon that has been doped with 5×10^{22} m^{-3} of boron atoms. Inasmuch as B is a group IIIA element in the periodic table (Figure 2.6) it acts as an acceptor in silicon. Thus, this material is p-type extrinsic, and it is necessary to use Equation 18.17, with p = 5×10^{22} m^{-3} since at room temperature all of the B acceptor impurities are ionized. The hole mobility, from Figure 18.18 at an impurity concentration of 5×10^{22} m^{-3}, is 0.028 m^2/V-s. Therefore, the conductivity is equal to

$$\sigma = p|e|\mu_h = (5 \times 10^{22} \text{ m}^{-3})(1.602 \times 10^{-19} \text{ C})(0.028 \text{ m}^2/\text{V}-\text{s}) = 224 \ (\Omega-\text{m})^{-1}$$

18.38 Calculate the room-temperature electrical conductivity of silicon that has been doped with 2×10^{23} m^{-3} of arsenic atoms.

Solution

Here we are asked to calculate the room-temperature electrical conductivity of silicon that has been doped with 2×10^{23} m^{-3} of arsenic atoms. Inasmuch as As is a group VA element in the periodic table (Figure 2.6) it acts as a donor in silicon. Thus, this material is n-type extrinsic, and it is necessary to use Equation 18.16, with $n = 2 \times 10^{23}$ m^{-3} since at room temperature all of the As donor impurities are ionized. The electron mobility, from Figure 18.18 at an impurity concentration of 2×10^{23} m^{-3}, is 0.05 m^2/V-s. Therefore, the conductivity is equal to

$$\sigma = n\,|e|\,\mu_e = \left(2 \times 10^{23}\ \text{m}^{-3}\right)\left(1.602 \times 10^{-19}\ \text{C}\right)\left(0.05\ \text{m}^2/\text{V-s}\right) = 1600\ (\Omega\text{-m})^{-1}$$

18.39 Estimate the electrical conductivity, at 125°C, of silicon that has been doped with 10^{23} m^{-3} of aluminum atoms.

Solution

In this problem we are to estimate the electrical conductivity, at 125°C, of silicon that has been doped with 10^{23} m^{-3} of aluminum atoms. Inasmuch as Al is a group IIIA element in the periodic table (Figure 2.6) it acts as an acceptor in silicon. Thus, this material is p-WSH H[WULQVLF DQG LW LV QHFHVVDU\ WR XVH (TXDWLRQ ¶←¶→ p in this expression is 10^{23} m^{-3} since at 125°C all of the Al acceptor impurities are ionized. The hole mobility is determined using Figure 18.19b. From the 10^{23} m^{-3} impurity concentration curve and at 125°C (398 K), μ_h = 0.017 m^2/V-s. Therefore, the conductivity is equal to

$$\sigma = p|e|\mu_h = (10^{23}\ \text{m}^{-3})(1.602 \times 10^{-19}\ \text{C})(0.017\ \text{m}^2/\text{V}-\text{s}) = 272\ (\Omega - \text{m})^{-1}$$

18.40 Estimate the electrical conductivity, at 85°C, of silicon that has been doped with 10^{20} m^{-3} of phosphorus atoms.

Solution

In this problem we are to estimate the electrical conductivity, at 85°C, of silicon that has been doped with 10^{20} m^{-3} of phosphorus atoms. Inasmuch as P is a group VA element in the periodic table (Figure 2.6) it acts as a donor in silicon. Thus, this material is n-WSH[WULQVLFDQGLWLVQHFHVVDU\ WRXVH(TXDWLRQ¶¶ n in this expression is 10^{20} m^{-3} since at 85°C all of the P donor impurities are ionized. The electron mobility is determined using Figure 18.19a. From the <10^{20} m^{-3} impurity concentration curve and at 85°C (358 K,) μ_e = 0.1 m^2/V-s. Therefore, the conductivity is equal to

$$\sigma = n\,|e|\,\mu_e = \left(10^{20}\ \mathrm{m^{-3}}\right)\left(1.602\times10^{-19}\ \mathrm{C}\right)\left(0.1\ \mathrm{m^2/V-s}\right) = 1.6\ \ (\Omega-\mathrm{m})^{-1}$$

The Hall Effect

18.41 Some hypothetical metal is known to have an electrical resistivity of 4×10^{-8} (Ω-m). Through a specimen of this metal that is 25 mm thick is passed a current of 30 A; when a magnetic field of 0.75 tesla is simultaneously imposed in a direction perpendicular to that of the current, a Hall voltage of -1.26×10^{-7} V is measured. Compute (a) the electron mobility for this metal, and (b) the number of free electrons per cubic meter.

Solution

(a) This portion of the problem calls for us to determine the electron mobility for some hypothetical metal using the Hall effect. This metal has an electrical resistivity of 4×10^{-8} (Ω-m), while the specimen thickness is 25 mm, I_x = 30 A and B_z = 0.75 tesla; under these circumstances, a Hall voltage of -1.26×10^{-7} V is measured. It is first necessary to convert resistivity to conductivity (Equation 18.4). Thus

$$\sigma = \frac{1}{\rho} = \frac{1}{4 \times 10^{-8} \ (\Omega - \text{m})} = 2.5 \times 10^{7} \ (\Omega\text{-m})^{-1}$$

The electron mobility μ_e may be determined using Equation 18.20b and upon incorporation of Equation 18.18, we have

$$\mu_e = |R_H|\sigma$$

$$= \frac{|V_H| d\,\sigma}{I_x B_z}$$

$$= \frac{\left(\left|-1.26 \times 10^{-7} \text{ V}\right|\right)(25 \times 10^{-3} \text{ m})\left[2.5 \times 10^{7} \ (\Omega - \text{m})^{-1}\right]}{(30 \text{ A})(0.75 \text{ tesla})}$$

$$= 0.0035 \ \text{m}^2/\text{V-s}$$

(b) Now we are to calculate the number of free electrons per cubic meter. From Equation 18.8 we have

$$n = \frac{\sigma}{|e|\,\mu_e}$$

$$= \frac{2.5 \times 10^{7} \ (\Omega\text{-m})^{-1}}{(1.602 \times 10^{-19} \text{ C})(0.0035 \ \text{m}^2/\text{V-s})}$$

$$= 4.46 \times 10^{28} \ \text{m}^{-3}$$

18.42 Some metal alloy is known to have electrical conductivity and electron mobility values of 1.5×10^7 $(\Omega\text{-m})^{-1}$ and 0.0020 m^2/V-s, respectively. Through a specimen of this alloy that is 35 mm thick is passed a current of 45 A. What magnetic field would need to be imposed to yield a Hall voltage of -1.0×10^{-7} V?

Solution

In this problem we are asked to determine the magnetic field required to produce a Hall voltage of -1.0×10^{-7} V, given that $\sigma = 1.5 \times 10^7$ $(\Omega\text{-m})^{-1}$, $\mu_e = 0.0020$ m^2/V-s, $I_x = 45$ A, and d = 35 mm. Combining Equations 18.18 and 18.20b, and after solving for B_z, we get

$$B_z = \frac{|V_H| d}{I_x R_H} = \frac{|V_H| \sigma d}{I_x \mu_e}$$

$$= \frac{\left(\left|-1.0 \times 10^{-7}\ \text{V}\right|\right)\left[1.5 \times 10^7\ (\Omega - \text{m})^{-1}\right]\left(35 \times 10^{-3}\ \text{m}\right)}{(45\ \text{A})\left(0.0020\ \ \text{m}^2/\text{V-s}\right)}$$

$$= 0.58\ \text{tesla}$$

Semiconducting Devices

18.43 Briefly describe electron and hole motions in a p–n junction for forward and reverse biases; then explain how these lead to rectification.

The explanations called for are found in Section 18.15.

18.44 How is the energy in the reaction described by Equation 18.21 dissipated?

Solution

The energy generated by the electron-hole annihilation reaction, Equation 18.21, is dissipated as heat.

18.45 What are the two functions that a transistor may perform in an electronic circuit?

Solution

In an electronic circuit, a transistor may be used to (1) amplify an electrical signal, and (2) act as a switching device in computers.

18.46 Cite the differences in operation and application for junction transistors and MOSFETs.

The differences in operation and application for junction transistors and MOSFETs are described in Section 18.15.

Conduction in Ionic Materials

18.47 We noted in Section 12.5 (Figure 12.22) that in FeO (wüstite), the iron ions can exist in both Fe^{2+} and Fe^{3+} states. The number of each of these ion types depends on temperature and the ambient oxygen pressure. Furthermore, we also noted that in order to retain electroneutrality, one Fe^{2+} vacancy will be created for every two Fe^{3+} ions that are formed; consequently, in order to reflect the existence of these vacancies the formula for wüstite is often represented as $Fe_{(1-x)}O$ where x is some small fraction less than unity.

In this nonstoichiometric $Fe_{(1-x)}O$ material, conduction is electronic, and, in fact, it behaves as a p-type semiconductor. That is, the Fe^{3+} ions act as electron acceptors, and it is relatively easy to excite an electron from the valence band into an Fe^{3+} acceptor state, with the formation of a hole. Determine the electrical conductivity of a specimen of wüstite that has a hole mobility of 1.0×10^{-5} m^2/V-s and for which the value of x is 0.060. Assume that the acceptor states are saturated (i.e., one hole exists for every Fe^{3+} ion). Wüstite has the sodium chloride crystal structure with a unit cell edge length of 0.437 nm.

Solution

We are asked in this problem to determine the electrical conductivity for the nonstoichiometric $Fe_{(1-x)}O$, given x = 0.060 and that the hole mobility is 1.0×10^{-5} m^2/V-s. It is first necessary to compute the number of vacancies per cubic meter for this material. For this determination let us use as our basis 10 unit cells. For the sodium chloride crystal structure there are four cations and four anions per unit cell. Thus, in ten unit cells of FeO there will normally be forty O^{2-} and forty Fe^{2+} ions. However, when x = 0.06, (0.06)(40) = 2.4 of the Fe^{2+} sites will be vacant. (Furthermore, there will be 4.8 Fe^{3+} ions in these ten unit cells inasmuch as two Fe^{3+} ions are created for every vacancy). Therefore, each unit cell will, on the average contain 0.24 vacancies. Now, the number of vacancies per cubic meter is just the number of vacancies per unit cell divided by the unit cell volume; this volume is just the unit cell edge length (0.437 nm) cubed. Thus

$$\frac{\#\text{ vacancies}}{\text{m}^3} = \frac{0.24 \text{ vacancies/unit cell}}{(0.437 \times 10^{-9}\text{ m})^3}$$

$$= 2.88 \times 10^{27} \text{ vacancies/m}^3$$

Inasmuch as it is assumed that the vacancies are saturated, the number of holes (p) is also 2.88×10^{27} m^{-3}. It is now possible, using Equation 18.17, to compute the electrical conductivity of this material as

$$\sigma = p|e|\mu_h$$

$$= (2.88 \times 10^{27}\text{ m}^{-3})(1.602 \times 10^{-19}\text{ C})(1.0 \times 10^{-5}\text{ m}^2/\text{V-s}) = 4613\ (\Omega\text{-m})^{-1}$$

18.48 At temperatures between 775°C (1048 K) and 1100°C (1373 K), the activation energy and preexponential for the diffusion coefficient of Fe^{2+} in FeO are 102,000 J/mol and 7.3 × 10^{-8} m^2/s, respectively. Compute the mobility for an Fe^{2+} ion at 1000°C (1273 K).

Solution

For this problem, we are given, for FeO, the activation energy (102,000 J/mol) and preexponential (7.3 × 10^{-8} m^2/s) for the diffusion coefficient of Fe^{2+} and are asked to compute the mobility for a Fe^{2+} ion at 1273 K. The mobility, $\mu_{Fe^{2+}}$, may be computed using Equation [illegible]; however, this expression also includes the diffusion coefficient $D_{Fe^{2+}}$, which is determined using Equation 5.8 as

$$D_{Fe^{2+}} = D_0 \exp\left(-\frac{Q_d}{RT}\right)$$

$$= (7.3 \times 10^{-8} \text{ m}^2/\text{s}) \exp\left[-\frac{102,000 \text{ J/mol}}{(8.31 \text{ J/mol-K})(1273 \text{ K})}\right]$$

$$= 4.74 \times 10^{-12} \text{ m}^2/\text{s}$$

Now solving for $\mu_{Fe^{2+}}$ yields

$$\mu_{Fe^{2+}} = \frac{n_{Fe^{2+}} e D_{Fe^{2+}}}{kT}$$

$$= \frac{(2)(1.602 \times 10^{-19} \text{ C/atom})(4.74 \times 10^{-12} \text{ m}^2/\text{s})}{(1.38 \times 10^{-23} \text{ J/atom-K})(1273 \text{ K})}$$

$$= 8.64 \times 10^{-11} \text{ m}^2/\text{V-s}$$

(Note: the value of $n_{Fe^{2+}}$ is two, inasmuch as two electrons are involved in the ionization of Fe to Fe^{2+}.)

Capacitance

18.49 A parallel-plate capacitor using a dielectric material having an ε_r of 2.5 has a plate spacing of 1 mm (0.04 in.). If another material having a dielectric constant of 4.0 is used and the capacitance is to be unchanged, what must be the new spacing between the plates?

Solution

We want to compute the plate spacing of a parallel-plate capacitor as the dielectric constant is increased form 2.5 to 4.0, while maintaining the capacitance constant. Combining Equations 18.26 and 18.27 yields

$$C = \frac{\varepsilon A}{l} = \frac{\varepsilon_r \varepsilon_0 A}{l}$$

Now, let us use the subscripts 1 and 2 to denote the initial and final states, respectively. Since $C_1 = C_2$, then

$$\frac{\varepsilon_{r1} \varepsilon_0 A}{l_1} = \frac{\varepsilon_{r2} \varepsilon_0 A}{l_2}$$

And, solving for l_2

$$l_2 = \frac{\varepsilon_{r2} l_1}{\varepsilon_{r1}} = \frac{(4.0)(1 \text{ mm})}{2.5} = 1.6 \text{ mm}$$

18.50 A parallel-plate capacitor with dimensions of 100 mm by 25 mm and a plate separation of 3 mm must have a minimum capacitance of 38 pF (3.8×10^{-11} F) when an ac potential of 500 V is applied at a frequency of 1 MHz. Which of those materials listed in Table 18.5 are possible candidates? Why?

Solution

This problem asks for us to ascertain which of the materials listed in Table 18.5 are candidates for a parallel-plate capacitor that has dimensions of 100 mm by 25 mm, a plate separation of 3 mm so as to have a minimum capacitance of 3.8×10^{-11} F, when an ac potential of 500 V is applied at 1 MHz. Upon combining Equations 18.26 and 18.27 and solving for the dielectric constant ε_r we get

$$\varepsilon_r = \frac{\varepsilon}{\varepsilon_0} = \frac{lC}{\varepsilon_0 A}$$

$$= \frac{(3 \times 10^{-3}\,\mathrm{m})(3.8 \times 10^{-11}\,\mathrm{F})}{(8.85 \times 10^{-12}\,\mathrm{F/m})(100 \times 10^{-3}\,\mathrm{m})(25 \times 10^{-3}\,\mathrm{m})}$$

$$= 5.15$$

Thus, the minimum value of ε_r to achieve the desired capacitance is 5.15 at 1 MHz. Of those materials listed in the table, titanate ceramics, mica, steatite, soda-lime glass, and porcelain are candidates.

18.51 Consider a parallel-plate capacitor having an area of 2500 mm^2 and a plate separation of 2 mm, and with a material of dielectric constant 4.0 positioned between the plates. (a) What is the capacitance of this capacitor? (b) Compute the electric field that must be applied for 8.0×10^{-9} C to be stored on each plate.

Solution

In this problem we are given, for a parallel-plate capacitor, its area (2500 mm^2), the plate separation (2 mm), and that a material having an ε_r of 4.0 is positioned between the plates.

(a) We are first asked to compute the capacitance. Combining Equations 18.26 and 18.27, and solving for C yields

$$C = \frac{\varepsilon A}{l} = \frac{\varepsilon_r \varepsilon_0 A}{l}$$

$$= \frac{(4.0)(8.85 \times 10^{-12}\ \text{F/m})(2500\ \text{mm}^2)(1\ \text{m}^2/10^6\ \text{mm}^2)}{2 \times 10^{-3}\ \text{m}}$$

$$= 4.43 \times 10^{-11}\ \text{F} = 44.3\ \text{pF}$$

(b) Now we are asked to compute the electric field that must be applied in order that 8×10^{-9} C be stored on each plate. First we need to solve for V in Equation 18.24 as

$$V = \frac{Q}{C} = \frac{8 \times 10^{-9}\ \text{C}}{4.43 \times 10^{-11}\ \text{F}} = 181\ \text{V}$$

The electric field E may QRZ EH GHWHUPLQHG XVLQJ (TXDWLRQ ¶+ WKXV

$$E = \frac{V}{l} = \frac{181\ \text{V}}{2 \times 10^{-3}\ \text{m}} = 9.1 \times 10^4\ \text{V/m}$$

18.52 In your own words, explain the mechanism by which charge storing capacity is increased by the insertion of a dielectric material within the plates of a capacitor.

This explanation is found in Section 18.19.

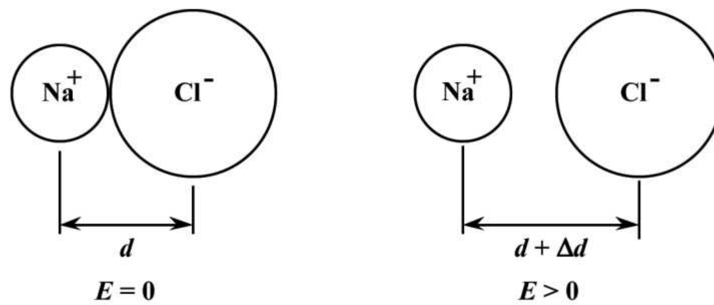
Field Vectors and Polarization

Types of Polarization

18.53 For NaCl, the ionic radii for Na^+ and Cl^- ions are 0.102 and 0.181 nm, respectively. If an externally applied electric field produces a 5% expansion of the lattice, compute the dipole moment for each Na^+–Cl^- pair. Assume that this material is completely unpolarized in the absence of an electric field.

Solution

Shown below are the relative positions of Na^+ and Cl^- ions, without and with an electric field present.



Now,

$$d = r_{Na^+} + r_{Cl^-} = 0.102 \text{ nm} + 0.181 \text{ nm} = 0.283 \text{ nm}$$

and

$$\Delta d = 0.05\, d = (0.05)(0.283 \text{ nm}) = 0.0142 \text{ nm} = 1.42 \times 10^{-11} \text{ m}$$

From Equation 18.28, the dipole moment, p, is just

$$p = q\,\Delta d$$

$$= (1.602 \times 10^{-19} \text{ C})(1.42 \times 10^{-11} \text{ m})$$

$$= 2.26 \times 10^{-30} \text{ C-m}$$

18.54 The polarization P of a dielectric material positioned within a parallel-plate capacitor is to be 1.0 × 10^{-6} C/m^2.

(a) What must be the dielectric constant if an electric field of 5 × 10^4 V/m is applied?

(b) What will be the dielectric displacement D?

Solution

(a) In order to solve for the dielectric constant in this problem, we must employ Equation 18.32, in which the polarization and the electric field are given. Solving for ε_r from this expression gives

$$\varepsilon_r = \frac{P}{\varepsilon_0 E} + 1$$

$$= \frac{1.0 \times 10^{-6} \text{ C/m}^2}{(8.85 \times 10^{-12} \text{ F/m})(5 \times 10^4 \text{ V/m})} + 1$$

$$= 3.26$$

(b) The dielectric displacement may be determined using Equation 18.31, as

$$D = \varepsilon_0 E + P$$

$$= (8.85 \times 10^{-12} \text{ F/m})(5 \times 10^4 \text{ V/m}) + 1.0 \times 10^{-6} \text{ C/m}^2$$

$$= 1.44 \times 10^{-6} \text{ C/m}^2$$

18.55 A charge of 3.5×10^{-11} C is to be stored on each plate of a parallel-plate capacitor having an area of 160 mm^2 (0.25 in.^{2}) and a plate separation of 3.5 mm (0.14 in.).

(a) What voltage is required if a material having a dielectric constant of 5.0 is positioned within the plates?

(b) What voltage would be required if a vacuum were used?

(c) What are the capacitances for parts (a) and (b)?

(d) Compute the dielectric displacement for part (a).

(e) Compute the polarization for part (a).

Solution

(a) We want to solve for the voltage when Q = 3.5×10^{-11} C, A = 160 mm^2, l = 3.5 mm, and ε_r = 5.0. Combining Equations 18.24, 18.26, and 18.27 yields

$$C = \frac{Q}{V} = \varepsilon \frac{A}{l} = \varepsilon_r \varepsilon_0 \frac{A}{l}$$

Or

$$\frac{Q}{V} = \varepsilon_r \varepsilon_0 \frac{A}{l}$$

And, solving for V, and incorporating values provided in the problem statement, leads to

$$V = \frac{Ql}{\varepsilon_r \varepsilon_0 A}$$

$$= \frac{(3.5 \times 10^{-11}\,\text{C})(3.5 \times 10^{-3}\,\text{m})}{(5.0)(8.85 \times 10^{-12}\,\text{F/m})(160\ \text{mm}^2)(1\ \text{m}^2/10^6\,\text{mm}^2)}$$

$$= 17.3\ \text{V}$$

(b) For this same capacitor, if a vacuum is used

$$V = \frac{Ql}{\varepsilon_0 A}$$

$$= \frac{(3.5 \times 10^{-11}\,\text{C})(3.5 \times 10^{-3}\,\text{m})}{(8.85 \times 10^{-12}\,\text{F/m})(160 \times 10^{-6}\,\text{m}^2)}$$

$$= 86.5 \text{ V}$$

(c) The capacitance for part (a) is just

$$C = \frac{Q}{V} = \frac{3.5 \times 10^{-11} \text{ C}}{17.3 \text{ V}} = 2.0 \times 10^{-12} \text{ F}$$

While for part (b)

$$C = \frac{Q}{V} = \frac{3.5 \times 10^{-11} \text{ C}}{86.5 \text{ V}} = 4.0 \times 10^{-13} \text{ F}$$

(d) The dielectric displacement may be computed by combining Equations 18.31, 18.32 and 18.6, as

$$D = \varepsilon_0 E + P = \varepsilon_0 E + \varepsilon_0(\varepsilon_r - 1)E = \varepsilon_0 \varepsilon_r E = \frac{\varepsilon_0 \varepsilon_r V}{l}$$

And incorporating values for ε_r and l provided in the problem statement, as well as the value of V computed in part (a)

$$D = \frac{(8.85 \times 10^{-12} \text{ F/m})(5.0)(17.3 \text{ V})}{3.5 \times 10^{-3} \text{ m}}$$

$$= 2.2 \times 10^{-7} \text{ C/m}^2$$

(e) The polarization is determined using Equations 18.32 and 18.6 as

$$P = \varepsilon_0(\varepsilon_r - 1)E = \varepsilon_0(\varepsilon_r - 1)\frac{V}{l}$$

$$= \frac{(8.85 \times 10^{-12} \text{ F/m})(5.0 - 1)(17.3 \text{ V})}{3.5 \times 10^{-3} \text{ m}}$$

$$= 1.75 \times 10^{-7} \text{ C/m}^2$$

18.56 (a) For each of the three types of polarization, briefly describe the mechanism by which dipoles are induced and/or oriented by the action of an applied electric field. (b) For solid lead titanate ($PbTiO_3$), gaseous neon, diamond, solid KCl, and liquid NH_3 what kind(s) of polarization is (are) possible? Why?

Solution

(a) For electronic polarization, the electric field causes a net displacement of the center of the negatively charged electron cloud relative to the positive nucleus. With ionic polarization, the cations and anions are displaced in opposite directions as a result of the application of an electric field. Orientation polarization is found in substances that possess permanent dipole moments; these dipole moments become aligned in the direction of the electric field.

(b) Electronic, ionic, and orientation polarizations would be observed in lead titanate. The lead, titanium, and oxygen would undoubtedly be largely ionic in character. Furthermore, orientation polarization is also possible inasmuch as permanent dipole moments may be induced in the same manner as for $BaTiO_3$ as shown in Figure 18.35.

Only electronic polarization is to be found in gaseous neon; being an inert gas, its atoms will not be ionized nor possess permanent dipole moments.

Only electronic polarization is to be found in solid diamond; this material does not have molecules with permanent dipole moments, nor is it an ionic material.

Both electronic and ionic polarizations will be found in solid KCl, since it is strongly ionic. In all probability, no permanent dipole moments will be found in this material.

Both electronic and orientation polarizations are found in liquid NH_3. The NH_3 molecules have permanent dipole moments that are easily oriented in the liquid state.

18.57 (a) Compute the magnitude of the dipole moment associated with each unit cell of $BaTiO_3$, as illustrated in Figure 18.35.

(b) Compute the maximum polarization that is possible for this material.

Solution

(a) This portion of the problem asks that we compute the magnitude of the dipole moment associated with each unit cell of $BaTiO_3$, which is illustrated in Figure 18.35. The dipole moment p is defined by Equation 18.28 as p = qd in which q is the magnitude of each dipole charge, and d is the distance of separation between the charges. Each Ti^{4+} ion has four units of charge associated with it, and thus $q = (4)(1.602 \times 10^{-19}\ C) = 6.41 \times 10^{-19}\ C$. Furthermore, d is the distance the Ti^{4+} ion has been displaced from the center of the unit cell, which is just 0.006 nm + 0.006 nm = 0.012 nm [Figure 18.35(b)]. Hence

$$p = qd = (6.41 \times 10^{-19}\ C)(0.012 \times 10^{-9}\ m)$$

$$= 7.69 \times 10^{-30}\ C\text{-}m$$

(b) Now it becomes necessary to compute the maximum polarization that is possible for this material. The maximum polarization will exist when the dipole moments of all unit cells are aligned in the same direction. Furthermore, it is computed by dividing the above value of p by the volume of each unit cell, which is equal to the product of three unit cell edge lengths, as shown in Figure 18.35. Thus

$$P = \frac{p}{V_C}$$

$$= \frac{7.69 \times 10^{-30}\ C-m}{(0.403 \times 10^{-9}\ m)(0.398 \times 10^{-9}\ m)(0.398 \times 10^{-9}\ m)}$$

$$= 0.121\ C/m^2$$

Frequency Dependence of the Dielectric Constant

18.58 The dielectric constant for a soda–lime glass measured at very high frequencies (on the order of 10^{15} Hz) is approximately 2.3. What fraction of the dielectric constant at relatively low frequencies (1 MHz) is attributed to ionic polarization? Neglect any orientation polarization contributions.

Solution

For this soda-lime glass, in order to compute the fraction of the dielectric constant at low frequencies that is attributed to ionic polarization, we must determine the ε_r within this low-frequency regime; such is tabulated in Table 18.5, and at 1 MHz its value is 6.9. Thus, this fraction is just

$$\text{fraction} = \frac{\varepsilon_r\,(\text{low}) - \varepsilon_r\,(\text{high})}{\varepsilon_r\,(\text{low})}$$

$$= \frac{6.9 - 2.3}{6.9} = 0.67$$

Ferroelectricity

18.59 Briefly explain why the ferroelectric behavior of $BaTiO_3$ ceases above its ferroelectric Curie temperature.

Solution

The ferroelectric behavior of $BaTiO_3$ ceases above its ferroelectric Curie temperature because the unit cell transforms from tetragonal geometry to cubic; thus, the Ti^{4+} is situated at the center of the cubic unit cell, there is no charge separation, and no net dipole moment.

DESIGN PROBLEMS

Electrical Resistivity of Metals

18.D1 A 95 wt% Pt-5 wt% Ni alloy is known to have an electrical resistivity of 2.35×10^{-7} Ω-m at room temperature (25°C). Calculate the composition of a platinum-nickel alloy that gives a room-temperature resistivity of 1.75×10^{-7} Ω-m. The room-temperature resistivity of pure platinum may be determined from the data LQ 7DEOH 7←7 DVVXPH WKDW SODWLQXP DQG QLFNHO IRUP D VROLG VROXWLRQ◀

Solution

This problem asks that we calculate the composition of a platinum-nickel alloy that has a room temperature resistivity of 1.75×10^{-7} Ω-m. The first thing to do is, using the 95 Pt-5 Ni resistivity data, determine the impurity contribution, and, from this result, calculate the constant A in Equation 18.11. Thus,

$$\rho_{total} = 2.35 \times 10^{-7} \ (\Omega\text{-m}) = \rho_i + \rho_t$$

From Table 18.1, for pure platinum, and using Equation 18.4

$$\rho_t = \frac{1}{\sigma} = \frac{1}{0.94 \times 10^{7} \ (\Omega-\text{m})^{-1}} = 1.064 \times 10^{-7} \ (\Omega\text{-m})$$

Thus, for the 95 Pt-5 Ni alloy

$$\rho_i = \rho_{total} - \rho_t = 2.35 \times 10^{-7} - 1.064 \times 10^{-7}$$

$$= 1.286 \times 10^{-7} \ (\Omega\text{-m})$$

In the problem statement, the impurity (i.e., nickel) concentration is expressed in weight percent. However, Equation 18.11 calls for concentration in atom fraction (i.e., atom percent divided by 100). Consequently, conversion from weight percent to atom fraction is necessary. (Note: we now choose to denote the atom fraction of nickel as c'_{Ni}, and the weight percents of Ni and Pt by C_{Ni} and C_{Pt}, respectively.) Using these notations, this conversion may be accomplished by using a modified form of Equation 4.6a as

$$c'_{Ni} = \frac{C'_{Ni}}{100} = \frac{C_{Ni} A_{Pt}}{C_{Ni} A_{Pt} + C_{Pt} A_{Ni}}$$

Here A_{Ni} and A_{Pt} denote the atomic weights of nickel and platinum (which values are 58.69 and 195.08 g/mol, respectively). Thus

$$c'_{Ni} = \frac{(5 \text{ wt\%})(195.08 \text{ g/mol})}{(5 \text{ wt\%})(195.08 \text{ g/mol}) + (95 \text{ wt\%})(58.69 \text{ g/mol})}$$

$$= 0.15$$

Now, solving for A in Equation 18.11

$$A = \frac{\rho_i}{c'_{Ni}\left(1 - c'_{Ni}\right)}$$

$$= \frac{1.286 \times 10^{-7} (\Omega - \text{m})}{(0.15)(1 - 0.15)} = 1.01 \times 10^{-6} (\Omega\text{ - m})$$

Now it is possible to compute the c'_{Ni} to give a room temperature resistivity of 1.75×10^{-7} Ω-m. Again, we must determine ρ_i as

$$\rho_i = \rho_{total} - \rho_t$$

$$= 1.75 \times 10^{-7} - 1.286 \times 10^{-7} = 4.64 \times 10^{-8} (\Omega\text{ - m})$$

If Equation 18.11 is expanded, then

$$\rho_i = Ac'_{Ni} - A c'^{2}_{Ni}$$

Or, rearranging this equation, we have

$$A c'^{2}_{Ni} - Ac'_{Ni} + \rho_i = 0$$

Now, solving for c'_{Ni} (using the quadratic equation solution)

$$c'_{Ni} = \frac{A \pm \sqrt{A^2 - 4A\rho_i}}{2A}$$

Again, from the above

$A = 1.01 \times 10^{-6}$ (Ω-m)

$\rho_i = 4.64 \times 10^{-8}$ (Ω-m)

which leads to

$$c'_{Ni} = \frac{1.01 \times 10^{-6} \pm \sqrt{(1.01 \times 10^{-6})^2 - (4)(1.01 \times 10^{-6})(4.64 \times 10^{-8})}}{(2)(1.01 \times 10^{-6})}$$

And, taking the negative root,

$$c'_{Ni} = 0.0483$$

Or, in terms of atom percent,

$$C'_{Ni} = 100\,c'_{Ni} = (100)(0.0483) = 4.83 \text{ at\%}$$

While the concentration of platinum is

$$C'_{Pt} = 100 - C'_{Ni} = 100.00 - 4.83 = 95.17 \text{ at\%}$$

Now, converting this composition to weight percent Ni, requires that we use Equation 4.7a as

$$C_{Ni} = \frac{C'_{Ni} A_{Ni}}{C'_{Ni} A_{Ni} + C'_{Pt} A_{PT}} \times 100$$

$$= \frac{(4.83 \text{ at\%})(58.69 \text{ g/mol})}{(4.83 \text{ at\%})(58.69 \text{ g/mol}) + (95.17 \text{ at\%})(195.08 \text{ g/mol})} \times 100$$

$$= 1.50 \text{ wt\%}$$

18.D2 Using information contained in Figures 18.8 and 18.38, determine the electrical conductivity of an 80 wt% Cu-20 wt% Zn alloy at -150°C (-240°F).

Solution

This problem asks that we determine the electrical conductivity of an 80 wt% Cu-20 wt% Zn alloy at –150°C using information contained in Figures 18.8 and 18.38. In order to solve this problem it is necessary to employ Equation 18.9 which is of the form

$$\rho_{total} = \rho_t + \rho_i$$

since it is assumed that the alloy is undeformed. Let us first determine the value of ρ_i at room temperature (25°C) which value will be independent of temperature. From Figure 18.8, at 25°C and for pure Cu, $\rho_t(25) = 1.75 \times 10^{-8}$ Ω-m. Now, since it is assumed that the curve in Figure 18.38 was generated also at room temperature, we may take ρ as $\rho_{total}(25)$ at 80 wt% Cu-20 wt% Zn which has a value of 5.3×10^{-8} Ω-m. Thus

$$\rho_i = \rho_{total}(25) - \rho_t(25)$$

$$= 5.3 \times 10^{-8}\ \Omega\text{-m} - 1.75 \times 10^{-8}\ \Omega\text{-m} = 3.55 \times 10^{-8}\ \Omega\text{-m}$$

Finally, we may determine the resistivity at –150°C, $\rho_{total}(-150)$, by taking the resistivity of pure Cu at –150°C from Figure 18.8, which gives us $\rho_t(-150) = 0.55 \times 10^{-8}$ Ω-m. Therefore

$$\rho_{total}(-150) = \rho_i + \rho_t(-150)$$

$$= 3.55 \times 10^{-8}\ \Omega\text{-m} + 0.55 \times 10^{-8}\ \Omega\text{-m} = 4.10 \times 10^{-8}\ \Omega\text{-m}$$

And, using Equation 18.4 the conductivity is calculated as

$$\sigma = \frac{1}{\rho} = \frac{1}{4.10 \times 10^{-8}\ \Omega - \text{m}} = 2.44 \times 10^{7}\ (\Omega\text{-m})^{-1}$$

18.D3 Is it possible to alloy copper with nickel to achieve a minimum tensile strength of 375 MPa (54,400 psi) and yet maintain an electrical conductivity of 2.5×10^6 $(\Omega\text{-m})^{-1}$? If not, why? If so, what concentration of nickel is required? You may want to consult Figure 7.16a.

Solution

To solve this problem, we want to consult Figures 7.16a and 18.9 in order to determine the Ni concentration ranges over which the tensile strength is greater than 375 MPa (54,500 psi) and the conductivity exceeds 2.5×10^6 $(\Omega\text{-m})^{-1}$.

From Figure 7.16a, a Ni concentration greater than about 30 wt% is necessary for a tensile strength in excess of 375 MPa. In Figure 18.9 is plotted the resistivity versus the Ni content. Since conductivity is the reciprocal of resistivity, the resistivity must be less than 40×10^{-8} Ω-m--i.e., $\frac{1}{2.5 \times 10^6\ (\Omega - \text{m})^{-1}}$. According to the figure, this will be the case for Ni concentrations less than 32.5 wt%.

Hence, it is possible to prepare an alloy meeting the criteria. The concentration of Ni would have to lie between about 30 and 32.5 wt%.

Extrinsic Semiconduction
Factors That Affect Carrier Mobility

18.D4 Specify an acceptor impurity type and concentration (in weight percent) that will produce a p-type silicon material having a room temperature electrical conductivity of 50 $(\Omega\text{-m})^{-1}$.

<u>Solution</u>

First of all, those elements which, when added to silicon render it p-type, lie one group to the left of silicon LQ WKH SHULRGLF WDEOH WKH\ LQFOXGH WKH JURXS ,,,$ HOHPHQWV) LJXUH --i.e., boron, aluminum, gallium, and indium.

Since this material is extrinsic and p-type, p >> n, and the electrical conductivity is a function of the hole concentration according to Equation 18.17. Also, the number of holes is about equal to the number of acceptor impurities, N_a. That is

$$p \sim N_a$$

From Equation 18.17, the conductivity is a function of both the hole concentration (p) and the hole mobility (μ_h). Furthermore, the room-temperature hole mobility is dependent on impurity concentration (Figure 18.18). One way to solve this problem is to use an iterative approach—i.e., assume some acceptor impurity concentration (which will also equal the value of p), then determine a "calculated" hole mobility from Equation 18.17—i.e.,

$$\mu_h = \frac{\sigma}{p\,|e|}$$

and, finally, compare this mobility with the "measured" value from Figure 18.18, taken at the assumed p (i.e., N_a) value.

Let us begin by assuming that $N_a = 10^{22}$ m^{-3}. Thus, the "calculated" mobility value is

$$\mu_h = \frac{\sigma}{p|e|} = \frac{50\ (\Omega - \text{m})^{-1}}{(10^{22}\ \text{m}^{-3})(1.602 \times 10^{-19}\ \text{C})} = 0.0312\ \text{m}^2/\text{V} - \text{s}$$

From Figure 18.18, at an impurity concentration of 10^{22} m^{-3} the "measured" hole mobility is 0.0362 m^2/V-s, which is slightly higher than the "calculated" value.

For our next choice, let us assume a lower impurity concentration, say 5×10^{21} m^{-3}. At this lower concentration there will be an increase of both "calculated" and "measured" electron mobilities. The "calculated" value is just

$$\mu_h = \frac{\sigma}{p\,|e|} = \frac{50\ (\Omega - \mathrm{m})^{-1}}{(5 \times 10^{21}\ \mathrm{m}^{-3})(1.602 \times 10^{-19}\ \mathrm{C})} = 0.0624\ \ \mathrm{m}^2/\mathrm{V-s}$$

Whereas, Figure 18.18 yields a "measured" μ_h of 0.0394 m^2/V-s, which is lower than the "calculated" value. Therefore, the correct impurity concentration will lie somewhere between 5×10^{21} and 10^{22} m^{-3} probably closer to the latter of these two values. At 8.0×10^{22} m^{-3}, both "measured" and "calculated" μ_h values are about equal (0.039 m^2/V-s).

It next becomes necessary to calculate the concentration of acceptor impurities in atom percent. This computation first requires the determination of the number of silicon atoms per cubic meter, N_{Si}, using Equation 4.2, which is as follows

$$N_{Si} = \frac{N_A \rho'_{Si}}{A_{Si}}$$

$$= \frac{(6.022 \times 10^{23}\ \mathrm{atoms/mol})(2.33\ \mathrm{g/cm^3})(10^6\ \mathrm{cm^3/m^3})}{28.09\ \mathrm{g/mol}}$$

$$= 5.0 \times 10^{28}\ \mathrm{m^{-3}}$$

(Note: in the above discussion, the density of silicon is represented by ρ'_{Si} in order to avoid confusion with resistivity, which is designated by ρ.)

The concentration of acceptor impurities in atom percent (C'_a) is just the ratio of N_a and ($N_a + N_{Si}$) multiplied by 100 as

$$C'_a = \frac{N_a}{N_a + N_{Si}} \times 100$$

$$= \frac{8.0 \times 10^{22}\ \mathrm{m^{-3}}}{(8.0 \times 10^{22}\ \mathrm{m^{-3}}) + (5.0 \times 10^{28}\ \mathrm{m^{-3}})} \times 100 = 1.6 \times 10^{-4}\ \mathrm{at\%}$$

Now, conversion to weight percent (C_a) is possible using Equation 4.7a as

$$C_a = \frac{C'_a A_a}{C'_a A_a + C'_{Si} A_{Si}} \times 100$$

where A_a and A_{Si} are the atomic weights of the acceptor and silicon, respectively. Thus, the concentration in weight percent will depend on the particular acceptor type. For example, for boron

$$C_B = \frac{C_B' A_B}{C_B' A_B + C_{Si}' A_{Si}} \times 100$$

$$= \frac{(1.6 \times 10^{-4} \text{ at\%})(10.81 \text{ g/mol})}{(1.6 \times 10^{-4} \text{ at\%})(10.81 \text{ g/mol}) + (99.99984 \text{ at\%})(28.09 \text{ g/mol})} \times 100$$

$$= 6.16 \times 10^{-5} \text{ wt\%}$$

Similar calculations may be carried out for the other possible acceptor impurities which yield

$$C_{Al} = 1.54 \times 10^{-4} \text{ wt\%}$$

$$C_{Ga} = 3.97 \times 10^{-4} \text{ wt\%}$$

$$C_{In} = 6.54 \times 10^{-4} \text{ wt\%}$$

18.D5 One integrated circuit design calls for diffusing boron into very high purity silicon at an elevated temperature. It is necessary that at a distance 0.2 μm from the surface of the silicon wafer, the room-temperature electrical conductivity be 1.2×10^3 $(\Omega\text{-m})^{-1}$. The concentration of B at the surface of the Si is maintained at a constant level of 1.0×10^{25} m^{-3}; furthermore, it is assumed that the concentration of B in the original Si material is negligible, and that at room temperature the boron atoms are saturated. Specify the temperature at which this diffusion heat treatment is to take place if the treatment time is to be one hour. The diffusion coefficient for the diffusion of B in Si is a function of temperature as

$$D(m^2/s) = 2.4 \times 10^{-4} \exp\left(-\frac{347 \text{ kJ/mol}}{RT}\right)$$

Solution

This problem asks for us to determine the temperature at which boron is to be diffused into high-purity silicon in order to achieve a room-temperature electrical conductivity of 1.2×10^3 $(\Omega\text{-m})^{-1}$ at a distance 0.2 μm from the surface if the B concentration at the surface is maintained at 1.0×10^{25} m^{-3}. It is first necessary for us to compute the hole concentration (since B is an acceptor in Si) at this 0.2 μm location.

From Equation 18.17, the conductivity is a function of both the hole concentration (p) and the hole mobility (μ_h). Furthermore, the room-temperature hole mobility is dependent on impurity concentration (Figure 18.18). One way to solve this problem is to use an iterative approach—i.e., assume some boron concentration, N_B (which will also equal the value of p), then determine a "calculated" hole mobility from Equation 18.17—i.e.,

$$\mu_h = \frac{\sigma}{p|e|}$$

and then compare this mobility with the "measured" value from Figure 18.18, taken at the assumed p (i.e., N_B).

Let us begin by assuming that $N_B = 10^{23}$ m^{-3}. Thus, the "calculated" mobility value is

$$\mu_h = \frac{\sigma}{p|e|} = \frac{1.2 \times 10^3 \ (\Omega - m)^{-1}}{(10^{23} \ m^{-3})(1.602 \times 10^{-19} \ C)} = 0.075 \ m^2/V-s$$

From Figure 18.18, at an impurity concentration of 10^{23} m^{-3} the "measured" hole mobility is 0.024 m^2/V-s, which is lower than the "calculated" value.

For our next choice, let us assume a higher boron concentration, say 10^{24} m^{-3}. At this higher concentration there will be a decrease of both "calculated" and "measured" hole mobilities. The "calculated" value is just

$$\mu_h = \frac{\sigma}{p|e|} = \frac{1.2 \times 10^3 \ (\Omega - m)^{-1}}{(10^{24} \ m^{-3})(1.602 \times 10^{-19} \ C)} = 0.0075 \ m^2/V-s$$

Whereas, Figure 18.18 yields a "measured" μ_h of 0.01 m^2/V-s, which is lower than the "calculated" value. Therefore, the correct impurity concentration will lie somewhere between 10^{23} and 10^{24} m^{-3}. At 6.0×10^{23} m^{-3}, "measured" and "calculated" values are about equal (0.0125 m^2/V-s).

With regard to diffusion, the problem is one involving the nonsteady-state diffusion of B into the Si, wherein we have to solve for temperature. Temperature is incorporated into the diffusion coefficient expression given in the problem. But we must first employ the solution to Fick's second law for constant surface composition ERXQGDU\ FRQGLWLRQV (TXDWLRQ LQ WKLV H[SUHVVLRQ C_0 is taken to be zero inasmuch as the problem stipulates that the initial boron concentration may be neglected. Thus,

$$\frac{C_x - C_0}{C_s - C_0} = 1 - \mathrm{erf}\left(\frac{x}{2\sqrt{Dt}}\right)$$

$$\frac{6.0 \times 10^{23}\,\mathrm{m}^{-3} - 0}{1.0 \times 10^{25}\,\mathrm{m}^{-3} - 0} = 1 - \mathrm{erf}\left(\frac{x}{2\sqrt{Dt}}\right)$$

which reduces to

$$0.9400 = \mathrm{erf}\left(\frac{x}{2\sqrt{Dt}}\right)$$

In order to solve this expression for a value $\frac{x}{2\sqrt{Dt}}$ of it is necessary to interpolate using data in Table 5.1. Thus

z	erf(z)
1.3	0.9340
z	0.9400
1.4	0.9523

$$\frac{z - 1.3}{1.4 - 1.3} = \frac{0.9400 - 0.9340}{0.9523 - 0.9340}$$

From which, z ZKLFK LV WR VD

$$1.3328 = \frac{x}{2\sqrt{Dt}}$$

Inasmuch as there are 3600 s/h (= t) and x = 0.2 μm (= 2 × 10^{-7} m) the above equation becomes

$$1.3328 = \frac{2 \times 10^{-7}\text{ m}}{2\sqrt{(D)(3600\text{ s})}}$$

which, when solving for the value of D, leads to

$$D = \frac{1}{3600\text{ s}}\left[\frac{2 \times 10^{-7}\text{ m}}{(2)(1.3328)}\right]^2 = 1.56 \times 10^{-18}\text{ m}^2/\text{s}$$

Now, equating this value to the expression for D given in the problem gives

$$D = 1.56 \times 10^{-18}\text{ m}^2/\text{s} = \left(2.4 \times 10^{-4}\right)\exp\left[-\frac{347{,}000\text{ J/mol}}{(8.31\text{ J/mol-K})(T)}\right]$$

To solve for T, let us take the natural logarithms of both sides of the above equation; this leads to

$$\ln\left(1.56 \times 10^{-18}\right) = \ln\left(2.4 \times 10^{-4}\right) - \frac{347{,}000}{8.31T}$$

$$-41.002 = -8.335 - \frac{4.176 \times 10^{4}}{T}$$

which yields a value for T of 1278 K (1005°C).

Conduction in Ionic Materials

18.D6 Problem 18.47 noted that FeO (wüstite) may behave as a semiconductor by virtue of the transformation of Fe^{2+} to Fe^{3+} and the creation of Fe^{2+} vacancies; the maintenance of electroneutrality requires that for every two Fe^{3+} ions, one vacancy is formed. The existence of these vacancies is reflected in the chemical formula of this nonstoichiometric wüstite as $Fe_{(1-x)}O$, where x is a small number having a value less than unity. The degree of nonstoichiometry (i.e., the value of x) may be varied by changing temperature and oxygen partial pressure. Compute the value of x that is required to produce an $Fe_{(1-x)}O$ material having a p-type electrical conductivity of 2000 $(\Omega\text{-m})^{-1}$ DVVXPH WKDW WKH KROH PRELOLW\ LV 1.0 $\times$ 10^{-5} m^2/V-s, the crystal structure for FeO is sodium chloride (with a unit cell edge length of 0.437 nm), and that the acceptor states are saturated.

Solution

This problem asks, for the nonstoichiometric $Fe_{(1-x)}O$, given the electrical conductivity [2000 $(\Omega\text{-m})^{-1}$] and hole mobility (1.0 $\times$ 10^{-5} m^2/V-s) that we determine the value of x. It is first necessary to compute the number of holes per unit volume (p) using Equation 18.17. Thus

$$p = \frac{\sigma}{|e|\mu_h}$$

$$= \frac{2000\ (\Omega - \text{m})^{-1}}{(1.602 \times 10^{-19}\ \text{C})(1.0 \times 10^{-5}\ \text{m}^2/\text{V-s})} = 1.25 \times 10^{27}\ \text{holes/m}^3$$

Inasmuch as it is assumed that the acceptor states are saturated, the number of vacancies is also 1.25×10^{27} m^{-3}. Next, it is possible to compute the number of vacancies per unit cell by taking the product of the number of vacancies per cubic meter times the volume of a unit cell. This volume is just the unit cell edge length (0.437 nm) cubed:

$$\frac{\#\ \text{vacancies}}{\text{unit cell}} = (1.25 \times 10^{27}\ \text{m}^{-3})(0.437 \times 10^{-9}\ \text{m})^3 = 0.10$$

A unit cell for the sodium chloride structure contains the equivalence of four cations and four anions. Thus, if we take as a basis for this problem 10 unit cells, there will be one vacancy, 40 O^{2-} ions, and 39 iron ions (since one of the iron sites is vacant). (It should also be noted that since two Fe^{3+} ions are created for each vacancy, that of the 39 iron ions, 37 of them are Fe^{2+} and 2 of them are Fe^{3+}). In order to find the value of $(1 - x)$ in the chemical formula, we just take the ratio of the number of total Fe ions (39) and the number of total Fe ion sites (40). Thus

$$(1 - x) = \frac{39}{40} = 0.975$$

Or the formula for this nonstoichiometric material is $Fe_{0.975}O$.

Semiconductor Devices

18.D7 One of the procedures in the production of integrated circuits is the formation of a thin insulating layer of SiO_2 on the surface of chips (see Figure 18.26). This is accomplished by oxidizing the surface of the silicon by subjecting it to an oxidizing atmosphere (i.e., gaseous oxygen or water vapor) at an elevated temperature. The rate of growth of the oxide film is parabolic—that is, the thickness of the oxide layer (x) is a function of time (t) according to the following equation:

$$x^2 = Bt \tag{18.37}$$

Here the parameter B is dependent on both temperature and the oxidizing atmosphere.

(a) For an atmosphere of O_2 at a pressure of 1 atm, the temperature dependence of B (in units of m^2/h) is as follows:

$$B = 800 \exp\left(-\frac{1.24 \text{ eV}}{kT}\right) \tag{18.38a}$$

where k is Boltmann's constant (8.62×10^{-5} eV/atom) and T is in K. Calculate the time required to grow an oxide layer (in an atmosphere of O_2) that is 75 nm thick at both 750°C and 900°C.

(b) In an atmosphere of H_2O (1 atm pressure), the expression for B (again in units of m^2/h) is

$$B = 215 \exp\left(-\frac{0.70 \text{ eV}}{kT}\right) \tag{18.38b}$$

Now calculate the time required to grow an oxide layer that is 75 nm thick (in an atmosphere of H_2O) at both 750°C and 900°C, and compare these times with those computed in part (a).

Solution

(a) In this portion of the problem we are asked to determine the time required to grow a layer of SiO_2 that is 75 nm (i.e., 0.075 μm) thick on the surface of a silicon chip at 900°C, in an atmosphere of O_2 (oxygen pressure = 1 atm). Thus, using Equation 18.37, it is necessary to solve for the time t. However, before this is possible, we must calculate the value of B from Equation 18.38a as follows:

$$B = 800 \exp\left(-\frac{1.24 \text{ eV}}{kT}\right) = (800) \exp\left[-\frac{1.24 \text{ eV}}{(8.62 \times 10^{-5} \text{ eV/atom-K})(900 + 273 \text{ K})}\right]$$

$$= 0.00378 \ \mu\text{m}^2/\text{h}$$

Now, solving for t from Equation 18.37 using the above value for B and that x = 0.075 μm, we have

$$t = \frac{x^2}{B} = \frac{(0.075\ \mu\text{m})^2}{0.00378\ \mu\text{m}^2/\text{h}}$$

$$= 1.49\ \text{h}$$

Repeating the computation for B at 750°C:

$$B = (800)\exp\left[-\frac{1.24\ \text{eV}}{(8.62 \times 10^{-5}\ \text{eV/atom-K})(750 + 273\ \text{K})}\right]$$

$$= 6.25 \times 10^{-4}\ \mu\text{m}^2/\text{h}$$

And solving for the oxidation time as above

$$t = \frac{(0.075\ \mu\text{m})^2}{6.25 \times 10^{-4}\ \mu\text{m}^2/\text{h}} = 9.0\ \text{h}$$

(b) This part of the problem asks for us to compute the heating times to form an oxide layer 75 nm thick at the same two temperatures (900°C and 750°C) when the atmosphere is water vapor (1 atm pressure). At 900°C, the value of B is determined using Equation 18.38b, as follows:

$$B = 215\exp\left(-\frac{0.70\ \text{eV}}{kT}\right) = (215)\exp\left[-\frac{0.70\ \text{eV}}{(8.62 \times 10^{-5}\ \text{eV/atom-K})(900 + 273\ \text{K})}\right]$$

$$= 0.212\ \mu\text{m}^2/\text{h}$$

And computation of the time t from the rearranged form of Equation 18.37, leads to

$$t = \frac{x^2}{B} = \frac{(0.075\ \mu\text{m})^2}{0.212\ \mu\text{m}^2/\text{h}}$$

$$= 0.0265\ \text{h} = 95.5\ \text{s}$$

And at 750°C, the value of B is

$$B = (215) \exp\left[-\frac{0.70\ \text{eV}}{(8.62 \times 10^{-5}\ \text{eV/atom-K})(750 + 273\ \text{K})}\right] = 0.0767\ \mu\text{m}^2/\text{h}$$

Whereas the time required to grow the 75 nm oxide layer is

$$t = \frac{x^2}{B} = \frac{(0.075\ \mu\text{m})^2}{0.0767\ \mu\text{m}^2/\text{h}}$$

$$= 0.073\ \text{h} = 264\ \text{s}$$

From the above computations, it is very apparent (1) that the 75 nm oxide layer forms more rapidly at 900°C (than at 750°C) in both O_2 and H_2O gaseous atmospheres, and (2) that the oxide layer formation is more rapid in water vapor than in oxygen.

18.D8 The base semiconducting material used in virtually all of our modern integrated circuits is silicon. However, silicon has some limitations and restrictions. Write an essay comparing the properties and applications (and/or potential applications) of silicon and gallium arsenide.

Solution

We are asked to compare silicon and gallium arsenide semiconductors relative to properties and applications.

The following are the characteristics and applications for Si: (1) being an elemental semiconductor, it is cheaper to grow in single-FU\VWDOOLQH IRUP ♂♀ EHFDXVH RI LWV HOHFWURQ EDQG VWUXFWXUH LW LV EHVW XVHG LQ WUDQVLVWRUV (3) electronic processes are relatively slow due to the low mobilities for electrons and holes (Table 18.3).

) RU * D$ V ♂¶♀ LW LV P XFK P RUH H[SHQVLYH WR SURGXFH LQDV P XFK DV LW LV D FRP SRXQG VHP LFRQGXFWRU ♂♀ because of its electron band structure it is best used in light-HP LWWLQJ GLRGHV DQG VHP LFRQGXFWLQJ ODVHUV ♂♀ LWV band J DS P D\ EH DOWHUHG E\ DOOR\ LQJ ♂♀ HOHFWURQLF SURFHVVHV DUH P RUH UDSLG WKDQ LQ 6L GXH WR WKH J UHDWHU P RELOLWLHV IRU HOHFWURQV DQG KROHV ♂♀ DEVRUSWLRQ RI HOHFWURP DJ QHWLF UDGLDWLRQ LV J UHDWHU LQ * D$ V DQG WKHUHIRUH WKLQQHU OD\ HUV DUH required for solar cells.